Human Development Index: PNG progress and a mathematical explanation

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Abstract

The Human Development Index attempts to measure human well-being and its development over time in multiple countries across the world. Relative values of this index seem to possess an undesirable inherent stability with little indication of the removal of inequality. Per-capita Gross National Income data is at least visibly consistent with a Lognormal probability distribution suggesting that poverty may be the result of multiplicatively interdependent factors. Thus there may be a certain inevitability that, without special intervention, the rich will become richer and the poor, poorer.

Keywords: Human Development Index, Gross National Income, probability distribution, frequency distribution, cumulative frequency distribution, lognormal distribution, law of proportionate effect.

Introduction

A probability distribution is a mathematical function of certain quantitative variables describing the probabilities of occurrence of various possible outcomes in an experiment. The probability or frequency distributions arising from summarizing large masses of raw data can be used to model the behavior of random variables whose characteristics are known. These variables emanate from various real world situations. When a particular distribution can be fitted to a set of empirical data, the distribution is commonly used to make predictions about probable future behavior of the system generating the data. However, the fitting can also be used to suggest assumptions about the origin or causes of the empirical data based on knowledge of characteristics of the variable giving rise to a particular distribution.

This paper (a further development and updating of Anderson, 2014) reviews some of the data recorded in the Human Development Reports (HDR) developed over the past three decades. It notes the relative progress of PNG and its near neighbours on the Human Development Index (HDI). The perceived lack of progress relative to more developed countries in the same region leads to the examination of one of the several factors, the per capita Gross National Income (GNI), from the perspective of its empirical data fit to the lognormal distribution. The assumption is made that if empirical random data from an entity can be fitted to a particular distribution, hypotheses may be established concerning the underlying natural or other causes of the behaviour of the entity.

Human Development Index (HDI)

The HDI is a composite statistic intended to be a holistic measure of human well-being calculated from data collected annually by the United Nations Development Program (UNDP) for each country in the world where data is available. The information compiled includes data on aspects of human and economic life such as life expectancy, achieved educational levels, and reduced maternal mortality rates, measures of poverty and health, all as indicators of standard of living. These measures of human well-being are combined with per capita GNI, a quantitative measure of national economic growth, to produce the HDI, a ranking index ranging from approximately 0.3 (the low human development group) to nearly 1 (the very high human development group) for advanced countries. As data is collected annually, changing levels of estimated human development or well-being can be tracked for the 191 countries for which data is available.

The UNDP in its 2010 Human Development Report had begun to use a new method of calculating the HDI (The Human Development Index: Wikipedia) as of 2010 onwards. The three indices used are;

1. Life Expectancy Index (LEI) = $\frac{LE-20}{85-20}$ where LEI is 1 when Life expectancy at birth is 85 and 0 when Life expectancy at birth is 20.

2. Education Index (EI) = $\frac{MYSI+EYSI}{2}$ where

   i) Mean Years of Schooling Index (MYSI) = $\frac{MYS}{15}$ given that fifteen is the projected maximum of this indicator for 2025.

   ii) Expected Years of Schooling Index (EYSI) = $\frac{EYS}{18}$ given that eighteen is equivalent to achieving a master's degree in most countries.

3. Income Index (II) = $\frac{\ln(GNIpc)-\ln(100)}{\ln(75{,}000)-\ln(100)}$ where II is 1 when GNI per capita is \$75,000 and 0 when GNI per capita is \$100.

4. The geometric mean of the three indices above is calculated to be the HDI. That is, $HDI = \sqrt[3]{LEI \cdot EI \cdot II}$.

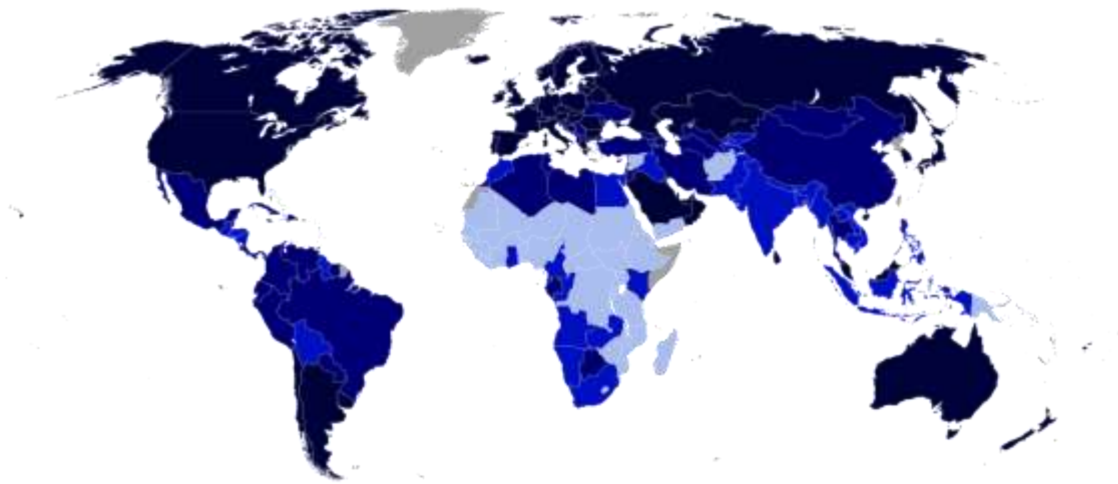


Figure 1 World map by quartiles of Human Development Index in 2018 (The Human Development Index: Wikipedia) showing the North (darker colours) South (lighter colours cutting a diagonal swathe from left to right) division and based on HDR (2018), Table 1, p 22.

The world map of Human Development Index (Figure 1) in 2018 (The Human Development Index: Wikipedia and based on HDR (2018), Table 1, p 22) identifies a general disparity in HDI values on a world map. The North (darker colours) and South (lighter colours cutting a diagonal swathe from left to right) division is apparent. Australia and New Zealand provide an interesting anomaly, being "high human development" countries in the far south and their relative geographical locations support the comparisons made in the paper.

The limitations of HDI, an index from easily measured quantities, as a measure of the quality of human life are readily acknowledged. "…. human well-being and freedom, and their connection with fairness and justice in the world, cannot be reduced simply to the measurement of GDP and its growth rate" (UNDP, p 24). Thus there is a need to avoid a reductionist approach which would equate human wellbeing completely with these easily measured indicators. Despite this acknowledged limitation, this paper assumes that the HDI data is still useful and proceeds to make best use of its availability.

In 2017, Papua New Guinea (PNG) was ranked 153 out of the 191 ranked countries and is classified as a country of "low human development" (Human Development Report, 2018, Table 1, p 22). Neighboring Solomon Islands (SI) was ranked 152, but still within the same low human development group. These rankings can be compared with those of Australia (rank 3) and New Zealand (rank 16), other near neighbors and sources of overseas aid for PNG who are ranked in the "very high development" group on the HDI. The disparity between these countries could hardly be much greater. PNG has, however, shown some limited improvement in HDI (Figure 2 and Table 1) with its HDI ranking growing from 0.380 (1990) to 0.544 (2017). Despite this upward trend, there has been a downward trend in growth rate (Figure 3) as measured over consecutive 10 year periods and as indicated by the decreasing slope of the plotted lines from 1990 to 2017.

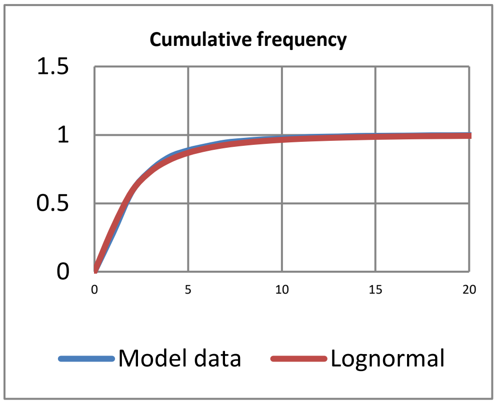


Figure 2 HDI Growth curves compared between selected countries in the Pacific region show little change in relative positions over time.

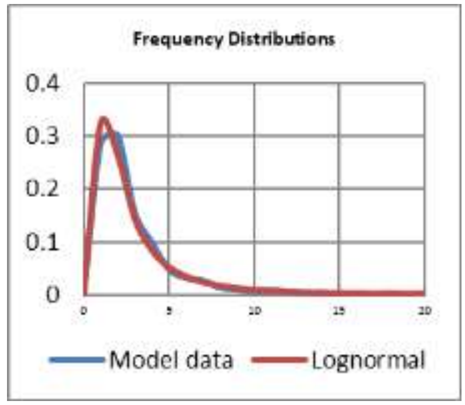


Figure 3 HDI differences compared as in Figure 2 showing little change in differences despite decades of overseas aid**.**

Possible factors influencing HDI

The hypothesis of this paper is motivated by the way, in which factors affecting HDI appear to be compounded as suggested in the HDR (Human Development Report 2013). The report notes that:

> "Environmental threats …. and natural disasters affect everyone, but they hurt poor countries and poor communities the most" (HDR Overview, p 6).

It is further noted that:

> "Although low HDI countries contribute least to global change, they are likely to endure the greatest loss in annual rainfall and sharpest increase in its variability with dire consequences for agricultural production and livelihoods" (HDR Overview, p 6).

Another issue that is also addressed is that:

> "Some poorer regions could benefit from a "demographic dividend", as the share of the working age population rises, but only if there is strong policy action" (HDR Overview, p 6).

These observations are consistent with the well-known observation that "the rich get richer and the poor get poorer" seemingly as a quite natural consequence of being where they are. These perceptions suggest that causative factors of HDI values may be multiplicative (Sarsoruo & Anderson, this volume) meaning that the value of a human development variable at any time is proportionate to its value at a previous period. Thus, a negative impact on a national economy will hurt poor counties more than those that are wealthy. If causative factors combine in such a multiplicative manner, the lognormal distribution suggests itself as a possible statistical model to fit the empirical data listed in the HDR.

Table 1 Growth in HDI values for selected neighbouring countries in the Pacific showing progressive relative development (HDR, 2018, Table 2 p28)**.** The data presented here has been recalculated according to the most recent method used as discussed in p2 of this paper where $HDI = \sqrt[3]{LEI \cdot EI \cdot II}$.

	1990	2000	2010	2012	2014	2015	2016	2017
NZ	0.818	0.869	0.899	0.905	0.910	0.914	0.915	0.917
Aus.	0.866	0.898	0.923	0.929	0.933	0.936	0.938	0.939
SI	N/A	0.450	0.507	0.529	0.539	0.546	0.543	0.546
PNG	0.380	0.449	0.520	0.530	0.536	0.542	0.543	0.544

HDI differences between these countries are quite stable (relatively flat plotted lines in Figure 3 and data in Table 2) showing little evidence of reduction of HDI disparity countries classified with “low human development” and their higher ranking neighbours despite decades of aid from the latter. This is here interpreted as suggesting that there might be other factors operating to produce these apparently stable disparities.

Table 2 Differences in HDI values for selected neighbouring countries in the Pacific showing only very small convergence of HDI values between neighbouring Pacific Island countries (calculated from data supplied in HDR, 2018, Table 2 p.28).

	1990	2000	2010	2012	2014	2016	2017
Aus-PNG	0.486	0.478	0.403	0.397	0.394	0.395	0.395
Aus-SI		0.448	0.416	0.400	0.394	0.395	0.393
SI-PNG		0.001	0.013	0.001	0.003	0.000	0.309
NZ-PNG	0.506	0.420	0.379	0.375	0.374	0.372	0.373

Modeling per capita GNI

World per capita GNI data for 1990 to 2017 are available online (Towards HDR 2019, UNDP: Human Development Reports) for consideration as lognormal distributions. Some summary data (Table 3) show the scale of variation in the countries discussed earlier to show disparities and relative locations of developing countries.

The GNI data sets also show the lognormal characteristic of a positively skewed distribution (Figures 4 & 5 for 1995 data and Figures 6 & 7 for 2011 data) consistent with outcomes resulting from multiplicative effects discussed in Simulating the Lognormal Distribution; A Monte Carlo method (Sarsoruo & Anderson, 2019) using the Law of Proportionate Effect. Actual values (red lines) from most of the 191 countries which have received a HDI ranking and for which GNI data was available, were sorted into 40 intervals chosen for optimum histogram display using Input Analyzer Utility. The total numbers of scores are shown in Table 3. Best fitting theoretical lognormal functions (blue lines) to the empirical data provide visible indication of goodness of fit. Both frequency functions (Figures 4 and 5) and cumulative frequency functions (Figures 6 and 7) provide reasonably confirming visibility tests for the claim of lognormal fitting to the GNI data.

Table 3 GNI data for 1990 to 2017 are compared for all counties for which data was available and for comparison between the countries previously discussed.

Year	Average	St. Dev.	N	Aus	NZ	SI	PNG
1990	$12439.99	$15718.99	188	$27790	$22089	$1614	$1867
1995	$12791.39	$17214.83	187	$29536	$23341	$1944	$2974
2000	$14395.8	$19327.59	189	$34536	$26135	$1495	$2694
2005	$15684.45	$19552.56	190	$37638	$29376	$1530	$2451
2010	$16386.17	$18855.47	191	$39920	$30530	$1399	$2872
2011	$16514.01	$18836.18	191	$40210	$31252	$1494	$2962
2012	$16704.84	$18629.07	191	$41486	$31995	$1797	$2998
2014	$17218.86	$18970.63	191	$42490	$32999	$1893	$3276
2016	$17739.06	$19417.67	191	$43637	$33679	$1850	$3398
2017	$17988.34	$19540.03	191	$43560	$33970	$1872	$3403

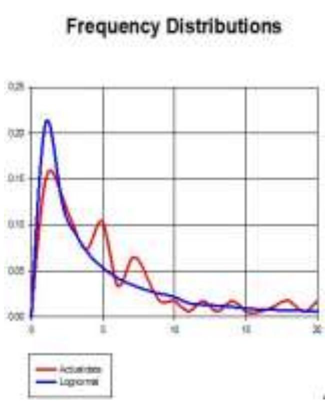


Figure 4 Frequency distribution of 1995 per capita GNI data with the blue curve indicating the empirical data and red the closest fit lognormal curve.

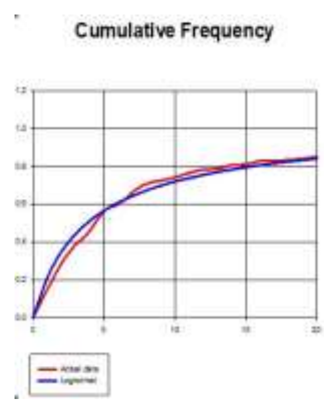
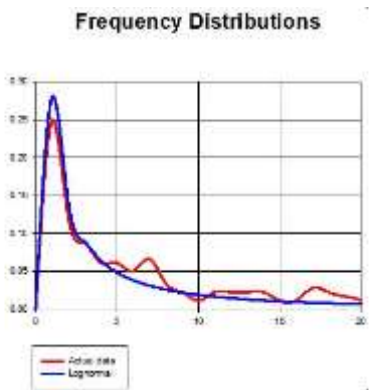


Figure 5 Corresponding cumulative Frequency distribution of 1995 per capita GNI data.

Comparison of the two sets of data (1995 & 2011), at least for the frequency (probability) functions, tends to suggest, at least from visibility, an improved fit for the 2011 data.



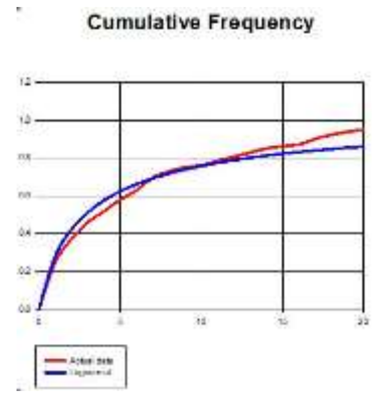


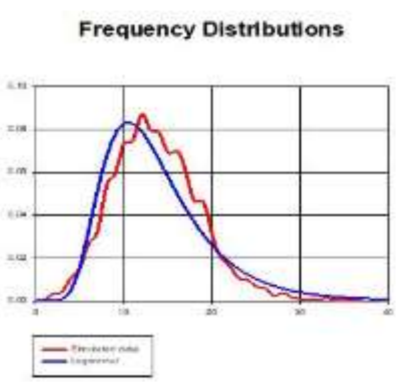
Figure 6 Frequency distribution of 2011 per capita GNI data with the blue curve indicating the empirical data and red the closest fit lognormal curve.

Figure 7 Corresponding Cumulative Frequency distribution of 2011 per capita GNI data.

Whilst the visibility tests provided so far might be, reasonably convincing, statistical tests are also available for more objective confirmation of any possible claims, which might be made for these distributions.

Other candidate distributions

The Input Analyzer display tool (discussed previously) can be used to further explain and show the relation between simulated data (red lines) and corresponding theoretical lognormal distributions (blue lines).



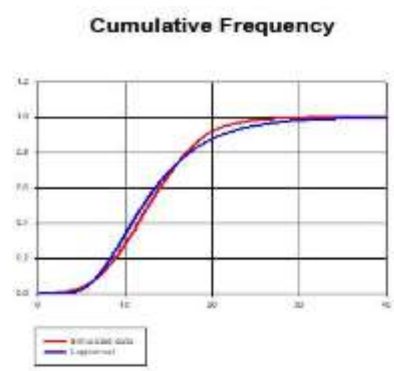


Figure 8 Comparative frequency distributions, simulated and theoretical, from a 5000 run simulation using data generated using the R script.

Figure 9 Comparative cumulative frequency distributions, simulated and theoretical, from a 5000 run simulation.

Table 4 Various asymmetric distributions fitted to simulated and GNI data with an error term (mean square error) indicating the closeness of fit**.** The results for the simulation data come from the 5000 run spreadsheet generated cases.

Simulation Function	Sq Error	1995 Function	Sq Error	2011 Function	Sq Error
Lognormal	0.00145	Wei-bull	0.00403	Beta	0.00538
Wei-bull	0.000581	Gamma	0.0115	Weibull	0.000462
Gamma	0.000359	Erlang	0.021	Lognormal	0.00882
Erlang	0.000366	Lognormal	0.00352	Gamma	0.0017
Beta	0.000444	Beta	0.019	Erlang	0.00679

The mean squared errors imply that the smaller the mean's squared error, the closer the line of best fit can be found. For example, comparing the lognormal distribution for the simulated data to the other distributions, the smallest mean square error appears to be 0.00352 which gives the line of best fit for the 1995 GNI data.

It needs to be acknowledged that there are numerous other statistical distributions (Hahn, & Shapiro, (1994)) which model positively skewed data such as the HDI data discussed in this paper. Relative degree of fittings of the data to candidate distributions can be estimated with a mean square error term (Sq Error in Table 4, with error terms generated by Input Analyzer referred to above).

Clearly, whilst the simulated data is best fitted with the lognormal distribution (Figures 7 & 8), there are other distribution functions which provide better fits to the HDI 1995 and 2011 empirical data than the lognormal despite the positive indications of the "visibility tests" referred to above**.** Thus, whilst the lognormal distribution may not provide the best fit, further tests can be applied to determine if the data is at least consistent with that distribution.

A second simulation (Sarsoruo & Anderson, 2019) was carried out using R programming (Kabacoff, 2011), an open source scripting language, to check if the data is at least consistent with the lognormal distribution providing the best fit. A script (see Appendix: R Source Code) was used to generate random incomes but this time the proportion variable (r_i) was drawn from a standard normal distribution (rather than the evenly distributed random distribution used with the Excel spreadsheet simulation (Sarsoruo, 2019). This variable is discrete, hence will take only positive values to show that the annual impact is proportionate to the Gross National Income for the last three decades from 1990 to 2017.

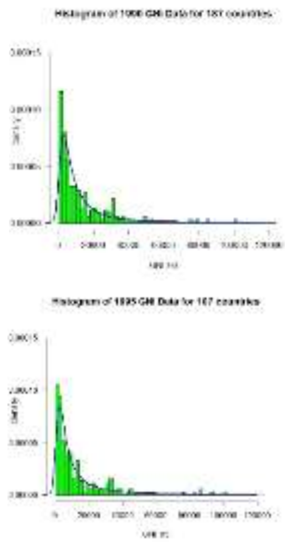


Figure 10 Lognormal distribution curves using simulated data against the GNI data for the years 1990 and 1995.

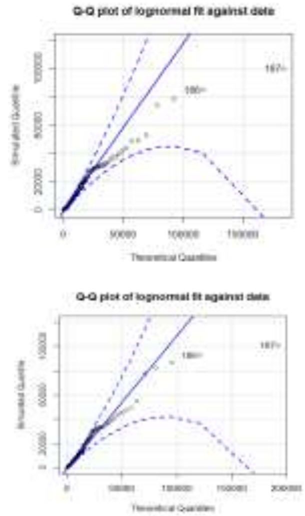


Figure 11 qqPlot of quantiles for simulated data (y axis) and theoretical distribution (x axis) shows initial points on the line, middle points below the line, whilst the end points are below and away from the blue 95% confidence interval lines.

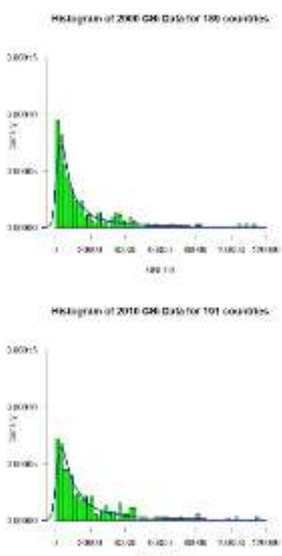


Figure 12 Lognormal distribution curves using simulated data against the GNI data for the years 2000 and 2010.

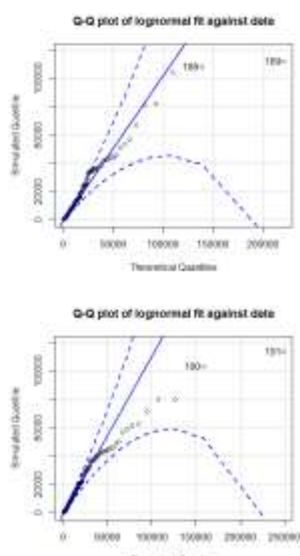


Figure 13 qqPlot of quantiles for simulated data (y axis) and theoretical distribution (x axis) shows initial points on the line, middle points below the line, whilst the end points are below and away from the blue 95% confidence interval lines.

The blue line in the histograms show that the simulated data is heavily skewed to the right. The Quantile-Quantile plots show the lognormal fit against data to model the flow of income. The fraction of income obtained for more than 180 countries rated as very high development to low development countries of the world for which data is available for the years 1990 and 1995 of the first decade (Figures 10 & 11) and the years 2000 and 2010 of the second decade (Figures 12& 13) .

An R script (Appendix: R Source Code) generated histograms for GNI data for the first two decades (Figures 10 & 12) with best fitting lognormal curves having tails that are positively skewed to the right. Obviously, these curves seem to show well-fitting overlay. However, to confirm these lognormal fits to the data the Quantile-Quantile plots (Figures 11&13) are used where two data sets are tested to see if they will produce the same distribution.

The plotted points which fall on the 45^o reference line implies that there is an equal density giving a good fit of incomes from 0 to $40,000 with most points lying between the 95% confidence lines(dotted) shown in blue. Further up, and below the 45^o line, from $40,000 to $80,000, the simulated data seem to become less dense. At the top right the simulated data seem to become denser. That is, two points representing GNI data of two different countries seem to have more extreme income earning values of between $80,000 to $120,000 with points lying below and away from the 95% confidence interval lines.

The distribution curves (Figures 10 & 12) discussed above shows the flow of income obtained across more than 180 countries rated as very high development to low development countries of the world for which data is available for the years (1990, 1995, 2000 and 2010) of the first and second decade as mentioned earlier in this paper. It is evident from the qqPlots (Figures 11 & 13) that the distribution of the GNI data seems to form an upward shape curve.

However, GNI data of a third decade (from 2012 to 2017) below (Figure 13) can be used again to show if the same distribution pattern (Figures 10 & 12) can be observed, so the hypothesis of this paper can be clearly supported.

The lognormal distribution of GNI data for the third decade, (Figure 14) generated from the same R script show best fitting lognormal curves. From the frequency histograms, the curves seem to give well-fitting overlays. Following the 45^o reference line, there seem to be an equal density giving a good fit of incomes from 0 to $40,000. From $40,000 to $80,000, the simulated data seem to become less dense implying that less than half of the 191 countries (HDR Report, 2018) stated in the HDR report obtained incomes of such value. At the furthest part, the simulated data seem to become denser. That is, from $90,000 to $140,000 two points (190,191) seem to lie away from the 95% confidence interval lines.

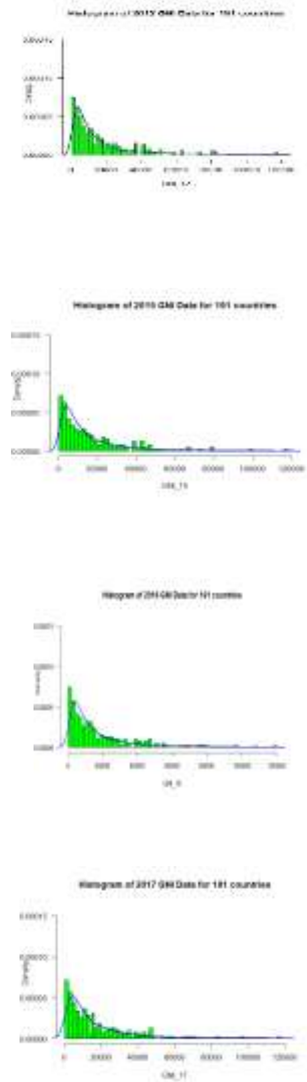


Figure 14 Lognormal distribution curves using simulated data against the GNI data for the years 2012 to 2017.

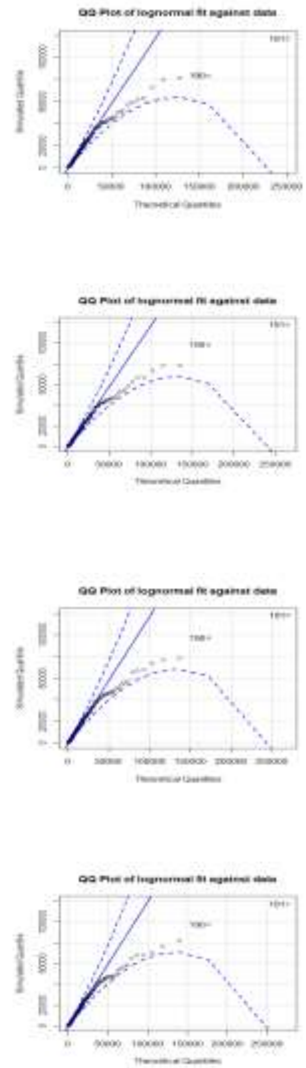


Figure 15 qqPlot of quantiles for simulated data (y axis) and theoretical distribution (x axis) shows initial points on the line, middle points below the line, whilst the end points are below and away from the blue 95% confidence interval lines.

Discussion of the QQ-Plots

The histograms (all even number Figures from 10 to 14) of the years in the last three decades (1990 to 2017) show that the curves are not symmetrical or uniformly distributed. All the curves have tails that are positively skewed heavily to the right. The qq-plots (all odd number Figures from 11 to 15) derived from the GNI simulated data (y axis) against the theoretical distribution (x-axis) suggest a common pattern. They show initial agreement with the law of proportionate effect giving rise to lognormal distributions, however, depart from the lognormal curves.

The countries that are causing the departure from the lognormal curves are countries that are ranked as very high human development countries in the HDR (2018) and oil rich countries. These countries are; Qatar (37), United Arab Emirates (34) and Kuwait (56) respectively. These countries are maintaining a consistent high economic rate over the last three decades. Qatar is the country represented by the point (191) that lies below and very far away from the 95% confidence lines. This implies that it has the highest per capita income in the world for the last three decades discussed in this paper, followed by Kuwait and United Arab Emirates. The countries that form the points that initially start a lognormal process are those countries identified as low human development countries in the HDR (2018). Some of these countries are landlocked countries. For example, Afghanistan ranked 168 out of 191 countries (HDR, 2018, Table 2 p28) is landlocked by the surrounding countries; Iran, Turkmenistan, Uzbekistan, Tajikistan, China and Pakistan.

Therefore, it is obvious that the empirical distribution shows a statistically significant (Anderson, 2014) difference from the theoretical distribution (Sarsoruo & Anderson, 2019). This can be further examined by the blue lines on the qq-plots that are 95% confidence limit. The points should plot close to the 45-degree reference line and within the 95% confidence limits.

For this reason, the hypothesis of this paper that empirical data is consistent with the theoretical distribution, as well as the many effects of the law of proportionate effects discussed by Sarsoruo & Anderson, (2019) are the factors that were found to be supported only by the “visibility tests” and the general observations (HDR Overview p6 quoted above) of the unequal effects of adverse conditions predicted on poor countries. Even so, note that the “visibility tests” discussed above has disadvantages such as skipping some random variables giving results that does not imply the real effects of data collected and analyzed.

Conclusion

The observed probability or frequency distributions do not seem to comply with the expected relationships of frequency curves given the underlying assumptions that resulted in such distributions in real systems. Identified physical and other characteristics in relation to an entity that attains random behavior can be modeled by a suitable choice of distribution function. In this instance, the empirical data on the entity HDI data is expected to fit a particular distribution, thus establishing the hypothesis underlying the factors causing the behavior of the entity. Several attempts were made to gain possible hypothesis in regard to the causes of empirically determined HDI data by fitting that data to several lognormal distributions. That is, the histograms exhibit visible agreement between the actual data and the lognormal theoretical curve.

The attempts at these data fitting were taken from careful studies of the data available in the HDR (2018) and (Towards HDR 2019, UNDP: Human Development Reports). The results show consistency depending on the multiplicative factors used to determine relative HDI values across most of the countries for which data is available. However, despite the fact that visibility data (Figures 4 to 9) generally support the hypothesis of this paper, more detailed and objective tests (all odd Figures 11 to 15) proved the later statement to be not true. The points in the qqPlots show initial agreement but then seem to depart from the lognormal curve.

Above all, there is a highly skewed distribution of measures of human well-being. Even with neighbouring countries, despite many years of aids from the highly developed to the low developed countries, there is still a big difference. This is a problem the world on a larger community has to solve. Without proper redistribution of wealth and improvement in other measures of human development well-being, the world will continue to experience the observation that the richer will become richer and the poor, poorer. Therefore, each nation has to seriously consider increasing its gross income earning from its commercial products to overcome factors that are affecting what at least has to be a picture of a lognormal process.

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Glossary

EI	Education Index
GNI	Per capita Gross National Income
HDR	Human Development Report
HDI	Human Development Index
II	Income Index
LEI	Life Expectancy Index
LE	Life expectancy at birth
MYS	Mean years of schooling (i.e. years that a person aged 25 or older has spent in formal education)
EYES	Expected years of schooling (i.e. total expected years of schooling for children under 18 years of age)
GNIpc	Gross national income at purchasing power parity per capita
PNG	Papua New Guinea
SI	Solomon Islands
UNDP	United Nations Development Program

Appendices

A. R Source Code

```
# Modelling GNI data available for years between 1990 and 2017
library(distr)
library(MASS)
library(car)
library(utils)
library(SweaveListingUtils)
library(vctrs)
library(readxl)
library(readr)

#Storing GNI data.
mydata<- read.csv (file = "GNI_17.csv", header=TRUE, Dec=",")
attach(mydata)
```

summary(GNI_17)
hist(GNI_17)

 # Fit the data in Data_95 to a lognormal distribution. Fitdistr provides Maximum-likelihood fitting (MLE and see note below) of univariate distributions to the simulated data. Estimates of meanlog and sdlog are also provided.
lnorm.fit <-fitdistr (GNI_17,"lognormal"

 # Find the mean and sd of a lognormal curve best fitting the simulated data: respectively meanlog, sdlog.
meanlog<-lnorm.fit$estimate["meanlog"]
sdlog<-lnorm.fit$estimate["sdlog"]
lnrv = rlnorm (10000, meanlog, sdlog)

 # Comparison with a theoretical lognormal distribution with this mean and this standard deviation: generate 10000 random lognormal variates from a distribution of the calculated mean = meanlog, sd = sdlog. Lnrv is the resulting probability density function.
lnrv = rlnorm (10000, meanlog, sdlog)

Display histogram and probability density function on a combined graph

```
hist (GNI_17, main="Histogram of 2017 GNI Data for 191 countries", border="black",
col="green",
    xlim=c (100,120000), ylim=c (0,0.00016), las= 1,
breaks=c (100,2000,4000,6000,8000,
            10000,12000,14000,15000,16000,18000,
            20000,22000,24000,26000,28000,
            30000,32000,34000,36000,38000,
            40000,42000,44000,46000,48000,
            50000,52000,54000,56000,58000,
            60000,62000,64000,66000,68000,
            70000,72000,74000,76000,778000,
            80000,82000,84000,86000,88000,
            90000,92000,94000,96000,98000,
            100000,101000,102000,103000,104000,106000,108000,
    110000,112000,114000,116000,118000,120000),
prob=TRUE)
```

 # provides histogram of simulation results in Data_95. Various options for histogram are available but not specified here
lines(density(lnrv), col="blue", lwd="2”) # **displays the best fitting probability density function**
QQplot of GNI data (2017) against theoretical lognormal distribution with specified mean and SD and with x and y axis labels specified.

```
qqPlot (GNI_17, dist= "lnorm”, meanlog=lnorm.fit$estimate["meanlog"],
sdlog=lnorm.fit$estimate["sdlog"],
       xlab="Theoretical Quantiles",
       ylab="Simulated Quantile”, ylim = c (100,120000))
```