ISSN 2706-8579

Electronic Journal of Informatics

Volume 4 December 2021
Divine Word University
Papua New Guinea

Electronic Journal of Informatics

Volume 4, December 2021
Divine Word University
Papua New Guinea

DWU Press Publishers
ISSN 2706-8579

Table of Contents

Electronic Journal of Informatics

The Electronic Journal of Informatics provides a platform for academics and non-academics to publish their research work. It provides an avenue for promoting a scholarly culture through research and exchange of ideas, experiences and insights for personal growth and professional development. The Journal is open for articles in areas including business and management, information and communications technology, mathematics and computing science, tourism and hospitality.

ISSN 2706-8579

Editors

Martin Daniel (PhD) is an Associate Professor and the Dean of the Faculty of Business & Informatics at DWU. He holds a Bachelor's degree in Information Technology from the University of Queensland in Australia, a Master's degree in Information Technology from the Queensland University of Technology in Australia and a Doctor of Philosophy from the Divine Word University. His interests include researching and modelling technological developments (e.g. e-government and e-learning) in PNG. Email: mdaniel@dwu.ac.pg and mdmartindaniel@gmail.com

Fiona N'Drower (PhD) is a lecturer in the Department of Tourism & Hospitality Management, Faculty of Business and Informatics at Divine Word University. Email: fndrower@dwu.ac.pg or fndrower@yahoo.com

Vanessa Uiari is a lecturer in the Department of Tourism and Hospitality Management. She has been employed with DWU since 8 February 2008. Her research interest is in critical tourism studies in the South Pacific and Oceania. She conducted an ethnographic study of the Isurava people of Iora Creek valley along the northern stretches of the Kokoda Trail in Oro province for her doctoral study, which she hopes to complete in 2020. Email: vuiari@dwu.ac.pg

Cyril Sarsoruo is a lecturer and Acting Head of the Department of Mathematics and Computing Science at DWU. He holds a Master's degree in Theoretical Mathematics and specializes in mathematical analysis. His research interests include functional equations and inequalities. Email: csarsoruo@dwu.ac.pg

Information for contributors

Articles should be 3000 – 6000 words in length including the abstract and references. Research notes, book reviews and editorial should be 900 – 1500 words. A no-more-than 100 words biographical note about the author should accompany the article. Full information for authors can be accessed at https://www.dwu.ac.pg/en/images/All_Attachements/FBI/e-Journals/E-Journal-guide-for-authors.pdf

Editorial note

Welcome to Volume 4 of the Electronic Journal of Informatics, which is an annual publication produced by the Faculty of Business and Informatics, Divine Word University. Publication of this journal began in 2019 to promote research culture through intensified research and knowledge exchange from its contributors.

Alcinda Trawen presents the lessons and insights from PNG tourism crisis management from the COVID-19 pandemic. She shows that communications, crisis management strategy and tourism specific support are essential elements for implementation to support crisis management in the tourism industry in Papua New Guinea.

Harry Gahare and **Fiona N'Drower** present the use of social capital as means of enhancing rural tourism from a PNG perspective. They show that communities/villages visited had in existence a key ingredient that supports tourism, which is the family and clan system that forms the basic foundation of traditional societies in PNG. The concept of social capital has prompted villages/communities to work together and promote tourism as a community initiative.

Martin Daniel presents the essential pillars of e-government for developing countries, around which e-government is developed, including citizen centricity, standardised infrastructures, back-office reorganisation, governance, new organisational models, social inclusion, people, processes, technology and resources. He provides some recommendations for developing countries so that these pillars are considered to shift from paper-based procedures to electronic processes, in the development of e-government.

Michaelyn Yaguro and **Raunu Gebo Sarsoruo** use an optimization model on current PNG Human Development Index data through an objective function to generate a maximized ranking for Papua New Guinea. The objective function is solved through linear programming with a maximized value between 0.7 to 0.9, thus placing PNG amongst the top very high human development countries and thereby achieving its' 2050 goal.

Peter K. Anderson presents the variation of the value of Pi on non-Euclidean surfaces. He shows that on spheres, Pi becomes smaller as circle circumferences grow larger with the reverse occurring on hyperbolic surfaces. This topic is of general interest, given the worldwide celebration of Pi day on 3/14/xx each year when we try to interest the general population, including students in Madang schools, colleges and universities in mathematics.

Ram Bilas Misra and **Ranjana Bajpai** present computing cube root of a real number. A method has been formulated to compute the real cube root of a real number irrespective of its nature whether containing only integral part, decimal part or both.

S. P. Khare and **Ram Bilas Misra** discuss primality and factorization - a computer challenge. They deal with the primality and factorization of RSA numbers including the latest test called the AKS Primality Test. Results of number theory are comprehensible by even the non-specialists but their proofs are most challenging.

Rik King discusses Benford's law, which concerns the prevalence of first and subsequent digits that appear in naturally occurring numerical transactions. He indicates that one possible forensic application is in the detection of fraud in machine-generated sets of data, which do not obey this law.

Hope that you find the articles interesting and informative.

Associate Professor Martin Daniel (PhD)
Coordinator and Chief Editor of the Journal

Papua New Guinea tourism crisis management insights: Lessons from COVID-19

Alcinda Trawen

Abstract

Tourism can show a considerable capacity to rebound after a crisis. However, in the short term, the impact of negative events on the sector can be significant. In light of the COVID-19 pandemic, the Papua New Guinea Tourism Promotion Authority undertook a study on the management of the pandemic by the tourism sector and the organizations' role in managing the impacts of COVID-19. The study employed a quantitative research method and used convenience sampling. Data were collected using survey questionnaires, facilitated by Survey Money. Statistical analysis was the approach taken to help draw meaning from the data, this was done with the help of Microsoft Excel. The study found that communications, crisis management strategy and tourism specific support are essential elements for implementation to support crisis management in the tourism industry in Papua New Guinea (PNG). Findings from this study add to the literature by offering a better understanding of tourism destination crisis management in PNG.

Keywords: crisis management strategy, communications plan, communication tools, safety and security, Papua New Guinea.

Introduction

A safe and secure place is an essential factor for a tourist destination (Beirman, 2016; Kordic et al., 2015). The tourism sector has a vested interest in securing travel and mitigating the impact of crises. The economic impact of natural disasters has increased at $335 billion in 2017 and the reach of epidemics has risen, draining $570 billion from the global economy (WTTC, 2019). The perceived level of safety and security is a key decision-making factor for travellers in selecting their destinations. Epidemics and outbreaks have become the new normal; with outbreaks such as H1N1 to the current COVID-19 estimated to have had a global economic impact between $45-$55 billion. The recovery time from disease cases took 19.4 months on average, with a range in recovery from 10 months to 34.9 months (WTTC, 2019).

The implications of crises increasingly require governments to engage with the private sector to improve crisis preparedness, management and recovery plans (PATA, 2020b). The effective management of a crisis requires the rapid activation of emergency plans as well as quick, accurate and transparent communication (PATA, 2020a). Responsiveness to ensure a speedy recovery should emphasize transparency, readiness and confidence; inviting the world back when ready, enticing visitors to return, building on a destination's tourist segments and rebuilding infrastructure strategically and smartly post-crisis (UNWTO, 2020).
Many countries have a national system for disaster risk reduction and disaster risk management, normally headed by a government department (ASEAN Secretariat, 2016).

Where crises affect tourism, these should link tourism-related stakeholders with civil defence and community response groups. Tourism should be integrated with existing systems, strategies and processes of national disaster risk management (COMSEC, 2017). After a crisis, quick actions for the recovery of the tourism sector can secure the employment and the life of community people who work for tourism and eventually lead to the recovery of the entire community from the crisis. A well-prepared tourism crisis management strategy can enable this.

Crisis tourism management in Papua New Guinea

PNG has had a reputation of being an unsafe destination, despite strong natural and cultural tourism products. PNG Tourism Promotion Authority (TPA), the national tourism organization has made significant efforts to communicate with the rest of the world that PNG is a safe destination. The main mandate of TPA is to promote PNG's many attractions to the rest of the world (PNGTPA, 2007). This has been done through various international and domestic market promotions and awareness.

Against the background of the COVID-19 pandemic, TPA undertook the development of a Crisis Management Strategy to identify, manage and mitigate crises or risks faced in the PNG tourism industry. In particular, sentiments from tourism stakeholders were considered important to determine the role of TPA in crisis management. This will assist the national tourism organization (NTO) to better prepare its' support to the PNG Tourism Industry to cope responsibly with crises and contribute to the sustainable and resilient development of the sector. An NTO can have a profound impact on how quickly and successfully the tourism industry can get back on its feet after a crisis event (Tourism Australia, 2020). Education, preparation, effective response to a crisis and management of the recovery process are critical for a tourism destinations' rapid recovery from a crisis event (ASEAN Secretariat, 2016).

Methodology

The Crisis/COVID-19 Management Survey provided the context to the development of a PNGTPA Crisis Management Strategy. The COVID-19 pandemic has motivated PNGTPA to be better prepared when faced with crises that may have consequences on the tourism industry. As a strategy by PNGTPA to assist in formulating a crisis management plan, a survey was conducted to allow tourism stakeholders to share their views and preferences on how PNGTPA can improve in its response towards crisis management. The survey comprised questions related to pre-crisis/COVID-19, crisis/COVID-19 response and crisis/COVID-19 recovery. The survey aimed to identify in pre-crisis, the tools the industry stakeholders need; to understand industry stakeholders' information needs in response to crisis/COVID19 and to identify what additional resources and support industry stakeholders might require during the recovery period in crisis management/COVID19.

Participants were given the opportunity to share their views from 5th to 23rd January 2021 using Survey Monkey, an online platform used for collecting feedback through a questionnaire. Links to the survey were distributed through email and social media on

PNGTPA channels inviting participants to complete the questionnaire. The responses received were from tourism private and public sectors that self-selected to participate in the survey and not from a structured sampling across tourism sectors. Additionally, paper-based surveys were distributed to participants at the PNGTPA Tourism Master Plan Validation Workshop, January 22nd, 2021. The survey approach used a non-random sample and therefore the results cannot perfectly extrapolate to the entire tourism industry. All results are shown in aggregate form. Statistical analysis in the form of Microsoft Excel (spreadsheet) was employed to categorise and understand the data that was collected for this study.

The survey yielded 112 responses. Of these, 61 were from the tourism industry. There also was a strong contingent of public sector participants related to the tourism industry who participated, comprising 51 participants. The respondents self-selected their sector type; accommodation, hospitality, transport, tour operations, tour wholesalers, attractions, travel agents and meetings, incentives, conferences and exhibitions (MICE) (Figure 1).

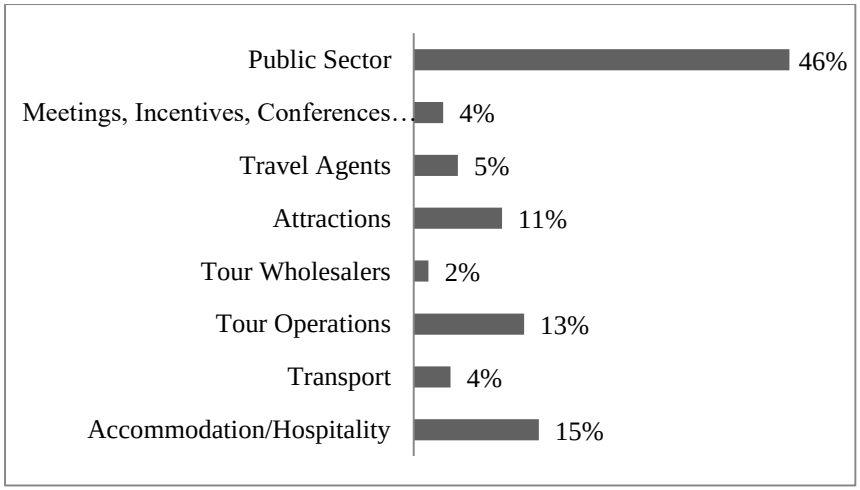


Figure 1: Sector of respondents

The profile of the respondents showed that half of the stakeholders were in the 25-39 demographic. In comparison, the public sector had the majority from the 25-39 demographic, industry stakeholders were mostly in the 40-59 demographic (Figure 2).

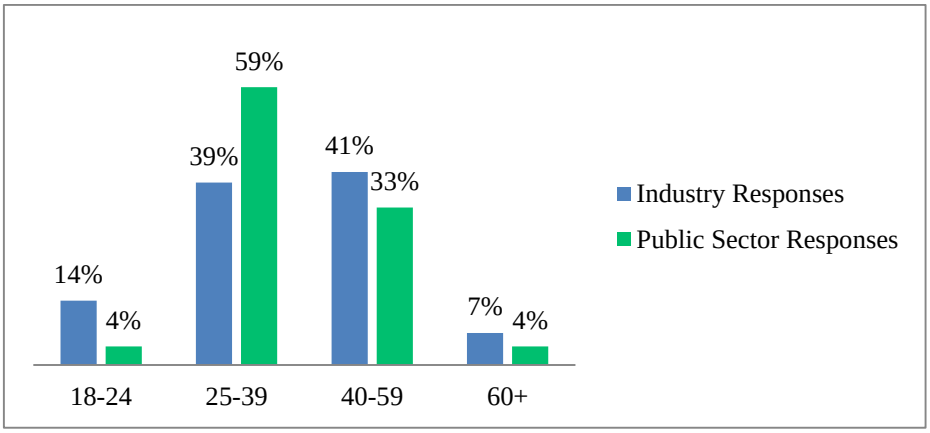


Figure 2: Age of respondents

Overall, there were more female responses with the private sector (55%) than the public sector, which had more male responses (55%). Moreover, half of the respondents were from the tourism private sector (54%) while 46 percent represented tourism public sector stakeholders.

Findings and discussion

The study found that communications, crisis management strategy and tourism specific support are essential elements to support crisis management in the PNG tourism industry. This will help TPA prepare its support to the industry to cope responsibly with crises and contribute to the sustainable and resilient development of the sector.

Key findings 1 – Communications

Many stakeholders did not receive tourism relevant information from TPA in the immediate response to COVID-19. The industry, in particular, received less information during the immediate response to the COVID-19 period compared to the public sector. The majority of tourism stakeholders were aware of the COVID-19 responses of the National Government. However, in terms of tourism specific efforts in COVID-19, stakeholders were more aware of information and initiatives from global tourism organizations. In terms of differences between the industry and public sector stakeholders concerning tourism efforts in the pandemic of national and provincial associations, the industry was less aware of associations tourism efforts compared to the public sector. There is a strong need for the NTO to support its stakeholders in crisis management.

Receiving tourism relevant information from TPA

Tourism stakeholders were asked if they received information from TPA during the COVID-19 pandemic. Half (51%) of respondents indicated that they received tourism relevant information from TPA prior to the pandemic. However, immediately after the pandemic, there were low levels (21%) of flow of tourism information from TPA. Likewise with tourism in recovery mode following vaccine development and some border relaxations, only 29% stated that they have been receiving tourism relevant information from TPA.

Results were further compared between industry and public sector. Industry responses on information provision from TPA before the pandemic were 51% whereas it was 46% from the public sector. Regarding the information provided in immediate response to the pandemic, there was a big difference from industry compared to the public sector (Figure 3). Likewise, in the recovery phase, 33% of the industry compared to 23% of the public sector claimed that they received relevant information.

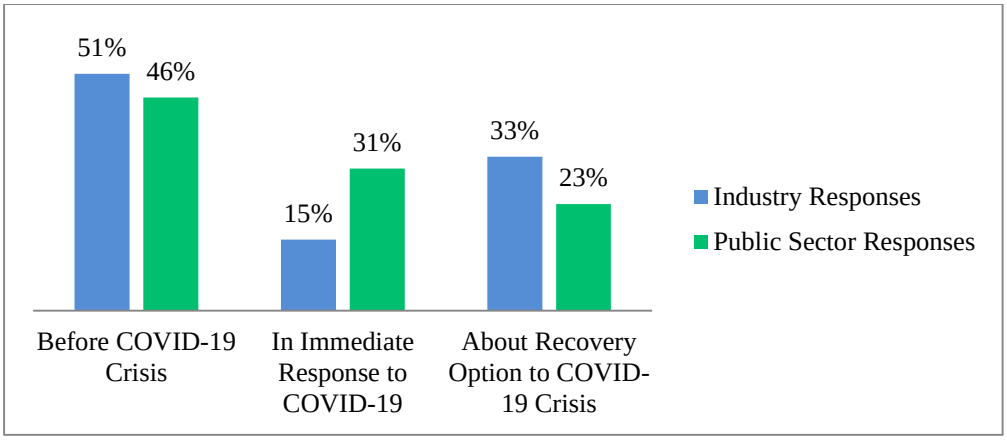


Figure 3: Receiving tourism relevant information from TPA

The results indicate some miscommunication and an opportunity to improve TPAs' communication and information procedures in crisis communication. Half of the respondents received information prior to the pandemic, however, immediately following the pandemic there was limited tourism information flowing to stakeholders. This could be attributed to the lack of a crisis management strategy, which would have detailed the appropriate responses to liaise with stakeholders.

Moreover, a lack of a coordinated communication plan and possibly a lack of information to pass on to the industry could be other contributing factors. Another dimension is capacity development; TPA personnel would need to be trained in the appropriate protocols to liaise with stakeholders in a time of crisis. Information in the recovery phase also needs to be improved from the significant low responses rate. More outreach is needed to the industry. In recovery, it seems more options could be improved to engage the public sector to support tourism industry recovery, likewise to increase information given to industry during recovery. Overall, TPA needs to increase its role in providing information and awareness in times of crisis. A communications plan that outlines protocols and information needed to communicate to stakeholders in a crisis is required including direction on human capacity development in this area. In COVID-19 recovery, perhaps thought to include an industry engagement plan with a focus on communications to reengage industry.

Awareness of the COVID-19 responses

A question was posed to ascertain the most popular group for tourism stakeholders where information flowed from the national government, provincial government, charities and non-governmental organizations, private sector and medical community. The majority (91%) of respondents was aware of the COVID-19 responses of the national government, followed by the medical community (55%), provincial government (52%) and private sector (51%). The least group where respondents were made aware of the COVID-19 responses were charities and non-governmental organizations (32%). The national government responses were more proactive and vocal in their reach. It could be attributed to different communication modes, their outreach or frequency of communication. Hence, the NTO could leverage the strength of national government channels to push tourism-related information and responses and utilize information from the national government to disseminate to industry. An important

consideration for TPA is to provide tourism information to relevant government stakeholders as part of the government engagement strategy.

Awareness of the tourism efforts

Respondents were more aware of efforts by global organizations (69%) such as World Tourism Organization (UNWTO), World Travel and Tourism Council (WTTC), International Air Travel Association (IATA) compared to TPA efforts (59%). Meanwhile, participants were less aware of the tourism efforts of regional organizations (19%) such as the Pacific Asia Travel Association (PATA) and Pacific Tourism Organization (SPTO), national and provincial tourism associations (19%) and provincial tourism offices and bureaus (17%).

The industry was more aware of global organizations tourism efforts compared to the public sector (Figure 4) while the public sector (18%) was more aware of regional associations tourism efforts compared to industry (14%). Also, the industry was less aware of the tourism efforts on a national scale from associations and provincial tourism offices than the public sector.

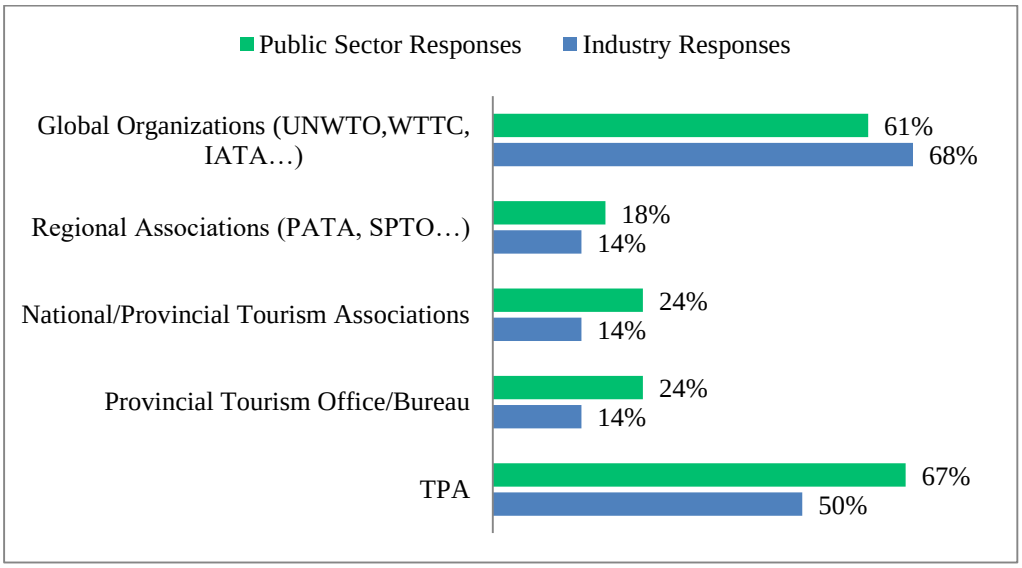


Figure 4: Awareness of tourism efforts

It is positive to note that respondents were aware of the tourism efforts of global entities and likewise on a national scale of TPA. However, it shows a significant disconnect in the outreach of tourism efforts in the provinces and national associations and regional entities. This disconnect could be due to a lack of efforts and initiatives or a lack of communication from these entities. Moreover, it could indicate that many industry stakeholders are not part of industry associations. There needs to be support for national and provincial tourism entities to increase their tourism information outreach. TPA could engage with industry associations to advance the outreach of their initiatives and membership, engage with provincial bureaus to advocate their information and disperse regional entities information to the industry.

TPA's role in crisis response

Respondents were asked to rank from the order of importance TPA's role in a crisis (Figure 5). The majority of stakeholders felt TPAs' main role is to provide links on industry and consumer websites to communicate up to date information about a crisis. The second role respondents felt as important is for TPA to communicate information to the media about visitor safety; this was followed by providing information about the destination including recovery progress. The area of less concern to respondents is in the role of responding to visitor enquiries about future bookings and cancellations.

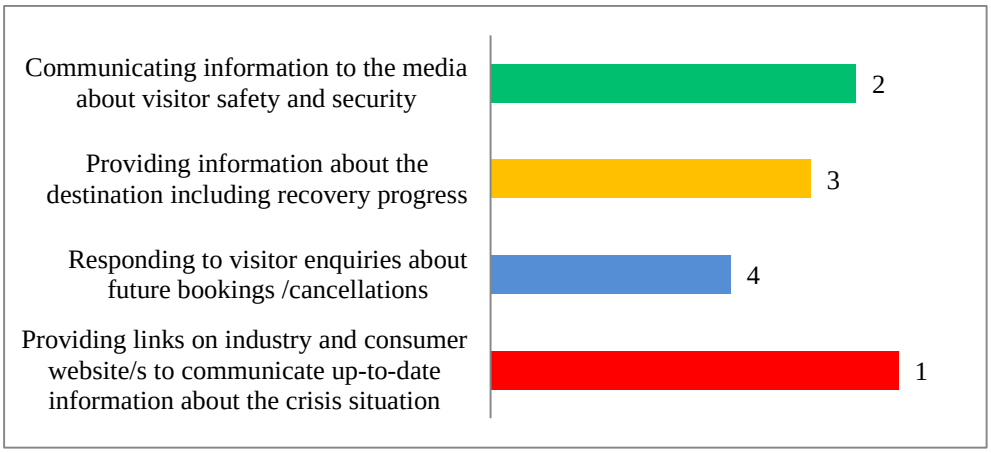


Figure 5: TPA role in crisis response

All stakeholders want TPA to primarily focus on providing links on industry and consumer websites to communicate up to date information about the crisis. Stakeholders need this information to assess and prepare their operations to manage a crisis. Stakeholders felt that the media plays an important role and expect TPA to work closely with them. In addition, as the industry moves towards coping with the pandemic, they are seeking information on recovery. The provision of a communication plan is essential for the implementation of information and awareness including up to date information. Any communication plan must include media engagement.

Available participatory forums or information to identify and meet your operation/organization needs in response to the crisis

The majority (74%) of respondents were not able to find any participatory forums or information to identify and meet their organizational needs in response to the crisis. While those that were to find (26%), did so through forums on social media as well as on global and local platforms and through online through various Websites.

There were many forums and platforms providing tourism relevant information which stakeholders were not aware of. They could have lacked the information to find these forums. Another possible consideration is that TPA was not aware of industry operational needs in the initial response to the pandemic to better support the industry with the relevant information they required. Moreover, for those that did find participatory spaces and information perhaps

TPA could look into partnering with or sharing information from these platforms. Further research needs to be conducted into industry operational needs post-pandemic.

Receiving crisis updates

Results show that there are preferences for the different modes of communication; social media (76%), email (73%), press release (63%) and website updates (59%). The next popular method of communication is newsletters (44%), meetings (30%) and videos (30%). In terms of other modes of communication, respondents mentioned the use of SMS, person to person, call centres and information booths.

Upon further analysis of responses from the industry and public sector, there was some variation in the preference of receiving crisis updates. The industry felt email communication was the most preferred method as opposed to the public sector preference for social media updates (Figure 6). In addition, another variation was in the importance of press releases, where the public sector (76%) felt that they were a more important tool for communication of updates as opposed to the industry (46%). Moreover, the industry felt a newsletter is a more popular form of communicating updates compared to the public sector.

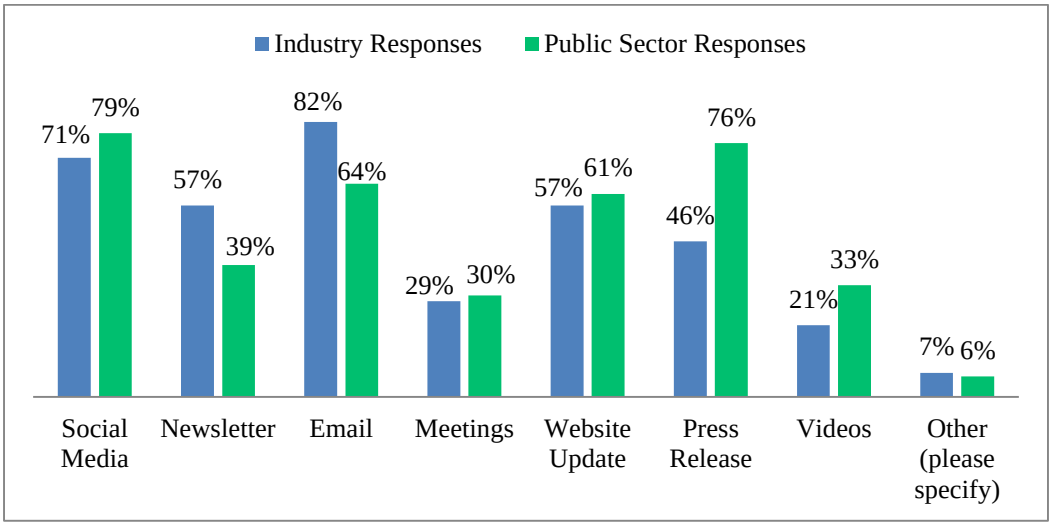


Figure 6: Receiving crisis updates

The general preference for social media means additional short real-time updated information provision is needed. Hence, TPA needs to focus some of its social media updates on crisis updates. Preference for email communication indicated that most industry operations regularly check emails. Moreover, the popularity of press releases as a mode of communication indicates that stakeholders want official statements from TPA. The variations in responses demonstrate that there need to be specific communication tools for specific sectors. For instance, the top four modes of communication the industry prefer is email, social media, newsletter and website updates as opposed to the public sector, which prefers social media, press releases, emails and website updates. TPA would need to update industry email listings as well as newsletter listings. Social media updates may consider including hashtag # on crisis updates to assess all crisis-related responses. Specific information can further be channelled to a COVID-19/crisis management page on the TPA website; likewise

ensure that templates on social media, email, website updates, press releases and newsletter templates are included in a TPA Crisis Communication Plan.

Key findings 2 – Crisis management strategy

From the start of the COVID-19 pandemic, many industry operators now have a risk management or crisis management plan. However, just over half of tourism operator respondents do not have a cancellation policy. All stakeholders advocated for the NTO to have a crisis management strategy.

Risk/crisis management plan

Tourism stakeholders were asked if they had a risk management or crisis management plan. This question lies at the heart of crisis management. Respondents (63%) stated that from COVID-19 their operations now had a risk management or crisis management plan (Figure 7). About 37% stated that their operations still do not have risk management or crisis management plan.

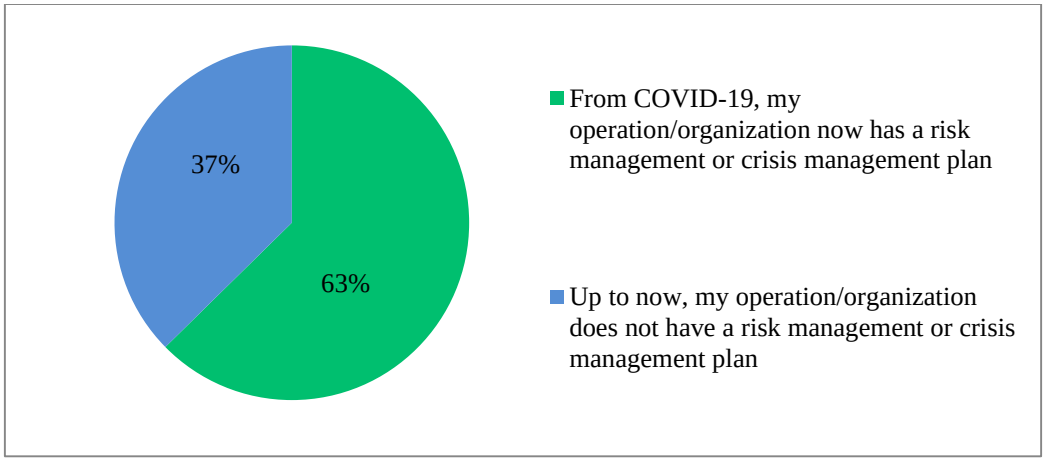


Figure 7: Risk/crisis management plan

All tourism operations need to have a risk management or crisis management plan to help identify the potential risks and seek a holistic approach in managing a crisis. The impact of the pandemic caught many operators unable to deal with the immediate closure of borders and the risks associated with the pandemic and how to manage it. There needs to be support to provide access to crisis management workshops or similar training. Likewise, the inclusion of crisis management tips on various communication channels such as websites, as well as regular information such as articles and case studies on aspects of crisis management is necessary.

Cancellation policy

In the event of a crisis, tourism businesses may be required to close such as through flight cancellations or access to their businesses may be closed such as roadblocks. This was the case in the recent global pandemic. In such cases, businesses are legally obliged to refund deposits for bookings that fall during that time. Hence, tourism businesses need to have reference to a cancellations policy or statement as part of their operations. From COVID-19,

less than half of respondents (47%) now have a cancellation policy (Figure 8) while 53% still did not have one.

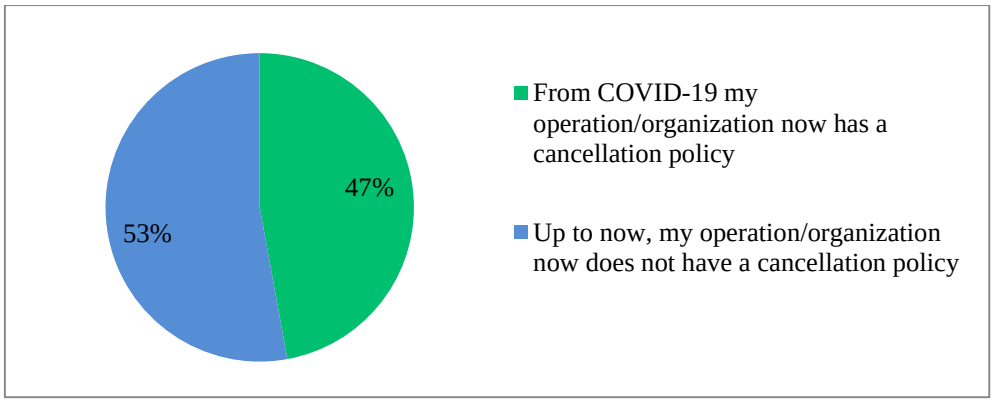


Figure 8: Cancellation policy

COVID-19 brought border closures and thus travel cancellations. There was also the closure of tourism sites and attractions, flight restrictions and postponement of festivals and events. The PNGTPA COVID-19 Tourism Industry Business Impact Survey from April 14th 2020, showed that 1,606 plus bookings were cancelled between March 2020 to January 2021. How businesses manage their cancellations may have an impact on visitors' perceptions and satisfaction with a business and consequently the wider country. Advocacy is needed, especially for local operators and tourism small to medium enterprises to have cancellation policies. It is important to encourage businesses to have a cancellation policy, which should clearly state the refund terms for a deposit and is communicated to tourists at the time of booking to mitigate any risks associated with future cancellations. Likewise, it is important to incorporate cancellation policies and contingencies for meetings, incentives, conferences and exhibitions (MICE) events such as popular cultural festivals. The responses indicate that additional work needs to be done to educate the industry about cancellation policies and contingencies.

Should TPA have a crisis management strategy?

Respondents overwhelmingly agreed that TPA should have a crisis management strategy. This question lies at the heart of the survey; where crises can cause significant challenges for the tourism industry and the need for TPA to adequately support the tourism stakeholders to identify, manage and mitigate crises or risks. Such a strategy will help TPA better prepare the industry to cope responsibly with crises and contribute through this means to the sustainable and resilient development of the tourism sector. It is recommended that TPA develop a crisis management strategy.

Key findings 3 -Tourism specific support

Stakeholders feel that TPA's main priority in crisis recovery should be in prioritizing destination communications in recovery efforts. The tourism industry in particular felt that the NTO should prioritize training and upskilling in recovery efforts compared to the public sector, which felt that the priority should be in domestic tourism support and health and

safety protocols. Most stakeholders expect more support and information from TPA than before the pandemic.

In recovery efforts what should TPA prioritize?

This question from the survey looks into the role of TPA in crisis recovery and asked respondents to share their views on where TPA should concentrate resources in recovery. The respondents were asked to choose only one priority area. Generally, the top areas of priority for respondents were destination communications (24%), health and safety protocols (22%), training and upskilling (21%) and domestic tourism support (19%). Lesser areas of priority were support incentives (6%), recovery marketing (4%) and travel facilitation (3%). Travel insurance support was an area of priority listed but respondents did not feel this was a priority.

A further comparison was undertaken between industry and public sector responses (Figure 9). There were some variances where the industry's main priority is training and upskilling (36%) compared to the public sector, which felt both domestic support (27%) and health and safety protocols (27%) were the main priority. However, both agreed that destination communications were the 2nd priority area. It is also interesting to note that the industry does not feel that travel facilitation is the main priority area, which could be alluded to the fact that this area is not within the TPA mandate. Considering the close results, it can be said that the main priorities are in the top four areas of destination communications, health and safety protocols, training and upskilling, and domestic tourism support.

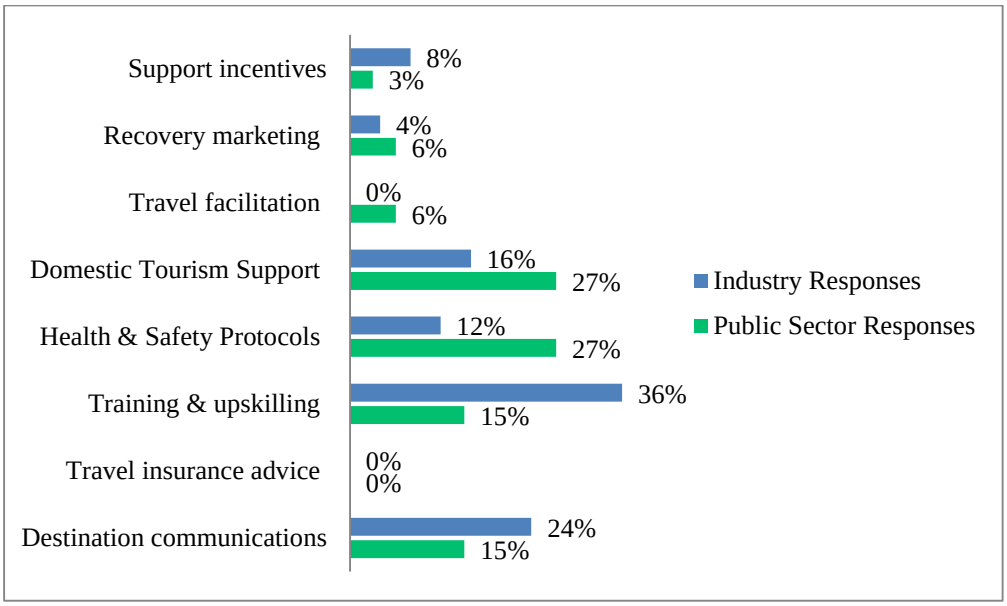


Figure 9: Tourism support

While domestic tourism support could encourage operators to develop, customize and promote domestic packages to revive travel within the local market, the development of destination communications plans, industry health and safety protocols, domestic tourism action plans, local product knowledge and digitization training initiatives and recovery marketing plan needs to be considered.

For the lesser areas of priority, TPA could consider working with tourism operators to ensure that they plan for a reduction in volume and greater personalization of service/new business models. The recovery marketing could work with operators to support marketing campaigns and marketing communication support. The respondents felt that travel facilitation to ensure an efficient and smooth opening of borders and travel insurance advice looking at supporting businesses with financial and legal advice are priority areas.

The nature of the pandemic and its unprecedented impact on tourism has meant that the industry is facing challenges in dealing with this unmatched event. In times of crisis affecting the tourism industry, the stakeholders expect more support from the NTO. To improve the expectations of stakeholders, more awareness of the role and responsibilities of NTO in crisis management should be done.

Human resource support in COVID-19

Stronger advocacy and promotion to mobilize resources and business advisory and information during times of crisis is required. The respondents want TPA to work with tourism operators to ensure that officially recommended protocols for health, hygiene and management of tourism operations are locally feasible and effective and support their implementation including initiatives, certification and waivers. In terms of training and upskilling, TPA could look into creating schemes and training to redistribute local product knowledge and skills and support innovation and digitization training initiatives.

The tourism sector needs immediate support to maintain employment. Due to a lack of resources, very few businesses are using this downtime to support the upskilling of their staff to improve the overall quality of tourism. TPA can liaise with the industry while staffs are idle to use this opportunity to work with universities, technical colleges, international organizations and training institutes to offer subsidized training programs. Consideration is given for mentoring services, especially for SMEs.

To support the burden on the tourism private sector employers, consideration to a program, supported by development partners, with specific interest in skills development, including certification, during the border closure would ensure consistency of standards across the sector. Likewise, when TPA creates healthy and safety guidelines for the tourism industry in COVID-19, support the industry is required to train their staff.

Recommendations

In light of the results from this study, the following actions are recommended for implementation to support crisis management in the PNG tourism industry from the experiences in managing the COVID-19 pandemic. Ultimately, this will lead to overall tourism destination improvement in building consumer confidence and maintaining markets in the overall PNG destination.

Communications campaign

Many stakeholders are unaware of the COVID-19 tourism information and initiatives to support their organizations. It is important to implement a clear and detailed communications program around the resources and guidance on application to enhance tourism business knowledge. TPA with relevant stakeholders could conduct outreach campaigns and provide vital information on COVID-19, its impact on the tourism sector, and on resources available on online platforms, with links to key statutory agencies and industry associations. This also provides an opportune time to utilize digital technology and advocate for its use by the industry as part of its recovery plans.

Provincial linkages

There are many operators in the provinces with little exposure to the information they need to support their operations. Additionally, as domestic tourism opens up, this will become an even larger and more urgent concern to ensure local operators are aware of the particulars of the pivot to domestic tourism. Domestic tourism is an opportunity for the industry to pilot and refine tourism offerings before international tourism resumes when borders are safely open. TPA should consider working with provincial offices and industry associations to identify appropriate linkages.

Crisis management strategy

Crisis communications is an essential element of a good crisis management system. It supports limiting the negative impact of a crisis by addressing the information needs of all industry stakeholders in an efficient, timely and responsible manner. A TPA crisis management strategy will support the organization in its roles and the lead up to, during, and after a crisis event. There needs to be support to provide access to crisis management workshops or similar training. Likewise, the inclusion of crisis management tips on various communication channels such as websites, as well as regular information such as articles and case studies on aspects of crisis management should be implemented. There needs to be provision for crisis management workshops or similar training that include elements on cancellation policies.

Crisis communications plan

A crisis management strategy should include a communication plan that outlines the protocols and necessary information to communicate to stakeholders in a crisis. TPA should increase its role in providing information and awareness. In COVID-19 recovery, an industry engagement plan with a focus on communications to reengage the industry should be included and engage the public sector in crisis recovery. TPA should participate in all government-wide forums, committees related to COVID-19 and crisis management, and provide tourism information to relevant government stakeholders as part of a holistic government engagement strategy. There needs to be support for national and provincial tourism entities to advance their tourism outreach. Further, engage with industry associations to advocate their information and membership to the wider industry and disseminate regional entities information to stakeholders.

Communication tools

The provision of a communication plan is essential for the delivery of information and awareness. Any communication plan must include media engagement. Further research needs to be conducted into industry needs in COVID-19 and crisis, promote and develop participatory forums for industry and improve communication and information access. TPA should consider updating its industry email listings as well as newsletter listings. TPA social media updates should consider including hashtag # on crisis updates to assess all crisis updates and responses with the TPA website to have COVID-19/crisis management page. Templates on social media, email, website updates, press releases and newsletter templates should be included in a TPA Crisis Communication Plan. Likewise, the provision of toolkits be included in crisis management strategy and for the provision of training in these tool kit areas.

Conclusion

The tourism sector has the potential to support the government in its effort towards COVID-19 recovery, especially PNG's long-term economic recovery and future growth. The tourism industry in COVID-19 is seeking advice on strategies to diversify and communicate. Support can be improved through close coordination by TPA, other relevant government agencies, development partners, provincial tourism officers and industry associations. Finally, the industry needs to ensure from a tourism perspective, that PNG is ready for a crisis so it can return to business as soon as possible.

References

ASEAN Secretariat (2016). *Tourism crisis communication manual*. ASEAN Secretariat: Jakarta, Indonesia.

Beirman, D. (2016). *Tourism risk, crisis and recovery management guide*. Council of Australian Tour Operators: University of Technology Sydney, Australia.

COMSEC (2017). *Risk and crisis management in tourism sector: Recovery from crisis in the oic member countries.* COMSEC Coordination Office: Ankara, Turkey.

Kordic, N., Stankoviic, K., Zivkovic, R., & Gajic, J. (2015). *Safety and security as factors of tourist destination competitiveness.* Retrieved from 25 August 2021, from https://www.researchgate.net/publication/283072429_Safety_and_Security_as_factors_of_tourist_destination_competitiveness

PATA (2020a). *Crisis communications planner.* PATA Head Office: Bangkok, Thailand.

PATA (2020b). *How to develop an effective post covid-19 communications strategy.* Retrieved 25 August 2021, from https://www.pata.org/calendar/how-to-develop-an-effective-post-covid-19-communications-strategy.

PNGTPA (2007). *Tourism Master Plan.* TPA: Port Moresby, Papua New Guinea

Tourism Australia (2020). *A guide to assist regional tourism organizations to prepare, respond and recover from a crisis.* Tourism Australia: Brisbane, Australia

UNWTO (2020). *Communicating crisis during and post COVID-19.* Retrieved 25 August 2021, from https://www.unwto.org/events/quo-vadis-tourism-unwto-communicating-crisis-pre-during-post-covid-19.

WTTC (2019). *Crisis Readiness.* World Travel and Tourism Council: London, United Kingdom

Acknowledgement

I would like to thank PNG Tourism Promotion Authority for supporting this study, particularly CEO Mr Eric Mossman, Ms Ingrid Kuman and Ms Mary Kanawi. Likewise, my sincere gratitude to Dr Fiona NDrower and Dr Martin Daniel for reviewing this paper. However, the responsibility for any errors of the fact of opinion remains with the author.

Author

Alcinda Trawen worked for UNWTO Regional Support Office for Asia and the Pacific, based in Japan. She was also formally with PNGTPA for thirteen years and prior to this a former lecturer within the Department of Tourism and Hospitality Management at DWU. She holds a Master's Degree in Tourism majoring in Ecotourism from James Cook University, Australia and a Master's Degree in Planning with Credit from the University of Otago, New Zealand.

Social capital a means of enhancing rural tourism: A Papua New Guinea perspective

Harry Gahare
Fiona N'Drower

Abstract

This study argues that social capital has always been part of Papua New Guinea's (PNG) cultural practices. This practice has encouraged the idea of rural tourism in PNG. For this study to be carried out, an indigenous research tool was employed to explore the value of social capital in empowering rural tourism. Twelve communities involved in tourism were invited to participate in this study to share their experiences and aspirations of tourism as an economic activity. They were encouraged to also share their thoughts on how tourism could be improved at their level. This study revealed that the communities/villages visited already had in existence a key ingredient that supports tourism. This is the family and clan system that forms the basic foundation of traditional societies in PNG. In rural PNG, social capital is an important asset that facilitates community survival and livelihood. The value of kinship is deeply rooted in PNG's traditional culture. Hence, the concept of social capital has prompted communities to work together and promote tourism as a community initiative.

Keywords: Social capital, rural tourism, indigenous research.

Introduction

The tourism industry is being embraced globally as an alternative to other industries in boosting the economy of destinations. Tourism allows people to travel and provide the opportunity to share resources. One motivation for tourism is typical - people from more advanced economies travel to underdeveloped economies for recreational purposes. The pull factors for the advanced economies are mainly, cultural experiences and natural wonders such as flora and fauna both on land and at sea. This paper aims to demonstrate that social capital is an essential ingredient for enhancing tourism in rural communities.

Background

This study is focused on Papua New Guinea (PNG) and in particular, local villages, where many ways are different from the urban areas. Having the luxury of electricity, clean running water through pipes, good road conditions and better health facilities are features associated with urban areas. These features are also important in supporting the growth of tourism in PNG. Tourism has been in PNG as early as 1884. The opportunity for tourism aroused when a shipping company, Burns Philp (BP), advertised a five-week trip to PNG via Thursday Island from Australia. Burns Philp had been given a mail delivery contract by the Australian administration to deliver mail throughout PNG. The company while delivering mail capitalized on the opportunity to include cruise tourism, as there was abundant space on the ship for passengers (Douglas, 1997). PNG was advertised as savage, exotic and primitive by

foreigners who had come to PNG for various reasons, as illustrated by the following quote from Burns Philp:

> Nowadays, when every 'Arry has done what not so many years ago was known as the 'Grand Tour', when alligator shooting on the Nile, lion hunting in Nubia, or tiger potting in the Punjab can be done by contract with Cook's tickets; when the Holy Land, Mecca or Khiva are all accessible to tourists; when every mountain in the Alps has been scaled, and even the Himalayas made the scene of mountaineering triumphs when shooting buffaloes in the 'Rockies' is almost as common as potting grouse on the moors; it comes almost with a sense of relief to visit a country really new, about which but little is known – a country of real cannibals and genuine savages... (Burns Philp, cited in Douglas (1997, p.56))

Drawing from the array of exotic adventures that one may have experienced with the kind of activity that is known in a particular area, PNG would have fallen into one of the categories given. Should an emphasis be on the natural beauty of the environment or the people? If little is known then, it is all the more, clear that neither the environment nor the people have much that is yet to be exposed. The niche that determines such resemblance is the relationship that exists between the environment and the people, not only in the natural circle but one that extends into the supernatural. Since little is known, the first experience would rather canvas not only the beauty but the reality as a tourist attraction. From the hind side, the inundated phenomena that may have captivated the attention would simply of tourists that in most instances portray an ideological juxtaposition. The latter reflects such an overarching perplexity by which certain significant items may not be overly exposed as a tourist destination, be it its people or the environment, Moreover, their coexistence is unimaginably enriching.

Literature review

Tourism

Despite tourism's relatively small contribution to PNG's economy compared to the extractive industries, tourism is promoted as a potential development strategy, especially in rural areas where there is much need for development since tourism can encourage sustainable development if managed well (PNGTPA, 2006). The PNG Tourism Promotion Authority (TPA) has since used different marketing slogans to promote PNG internationally including PNG as the land of the unexpected (Fitzpatrick, 2011) and currently as the land of a million different journeys (https://www.papuanewguinea.travel/corporate-site). To encourage tourism, cultural festivals are also marked as national events on the calendar, which enforces community cooperation (Whitford & Dunn, 2014). Cruise tourism is another tourism sector that PNG has embarked on as a platform to increase its revenue. However, Gibson (2012) warns that governments encouraging tourism ventures to be owned by local indigenous people must also empower them with the skills and knowledge to effectively manage their tourism affairs.

Social Capital

Humans have had to collaborate to survive for as long as they have been around (Pretty & Ward, 2001). Social capital has a relational dimension (Nahapiet & Ghoshal, 1998) consisting of ongoing relationships and reciprocal behaviours. Social capital is effective when

there is trust, reciprocity and cooperation (Flora, 1998). (Nahapiet & Ghoshal, 1998) argue that because social capital has a relational dimension it makes it possible for members of communities to support each other when engaging in business opportunities where cooperation is an essential part of social capital (Davidsson & Honig, 2003). Social capital has influenced tourism as an economic activity in peripheral communities (Zhao, Ritchie & Echtner, 2011). Bourdieu (1986) identified the collective use of human efforts in organizing and using human labour in obtaining and accumulating resources either for economic gain or mere survival as a form of social capital.

The PNG culture enforces one asset that is often overlooked, social capital. This asset plays a role in facilitating successful community outcomes (Diedrich, Benham, Pandihau & Sheaves, 2019). PNG has survived as a society because of the *wantok* system and embraces it as a tool for progression and survival, which is a form of social capital. Franklin (2007) also speaks of Melanesian values:

- The value of the clan (haus lain/wantok) – the value of maintaining relationships within clans and tribes in PNG is vital for survival. It is a form of social security and a foundation for social capital within rural communities, which means that understanding tourism development requires an understanding of the clan and tribal networks.
- The value of reciprocity (bekim, bekim bek) – the concept of reciprocity is closely linked to the value of clan and translates into an ethic of helping one another in times of need and returning the favour.
- The value of food (*kaikai*, *mumu*) – the sharing of food is central to the PNG culture - being hospitable to visitors, families and friends as a token of friendship, appreciation and goodwill.
- The value of ancestors (*tumbuna*, *tambaran*) – acknowledging ancestors guides how people within a village relate to each other. Either they could be descendants of the same ancestor or their ancestors are related.
- The value of ritual (*taboo*, *singsing*, *lotu*) - involves the preparation leading to an event or ceremony and sets out the actions that must be performed by those who initiate and those who participate in an event so the desired outcomes of the event will be achieved.
- The value of leadership (*hetman*) – an important element in rural communities is respecting and acknowledging the local leadership.
- The value of education (*skul*) – in contemporary PNG, there is a belief that education can lead to better and improved ways of living.
- The value of compensation (*peibek*, *bekim*, *birua*) – compensation in the PNG context refers to giving some form of payment to someone or a group of people after causing grievance to them. The aim is to maintain peace so there are no tribal fights or ethnic clashes, which usually end in bloody situations.
- The value of work (*wok*) – refers to the importance of putting in effort and making sacrifices to achieve one's goal.

The Melanesian values as described by Franklin (2007) form the core fundamentals of PNG's cultural fabric. It is through such values that the essence of social capital within the PNG context stems from. The trait of lending a helping hand to a family member, friend or even a stranger is expected not only by a clan but also by the society at large.

Rural tourism

Tourists travel to non-urban areas to participate in recreational activities. Some travel to enhance the locals' livelihood through tourism and preserve the host community's physical environment and cultural heritage (Lane, 1994). Rural tourism is also being used to alleviate poverty in underdeveloped economies (Sharpley & Sharpley, 1997) because of the diverse culture and natural environment in rural areas that make them attractive destinations (Nair et al., 2015). Over the years, tourism has also been seen as an agent for rural development (Saarinen & Lenao 2014) providing employment, better road links and supporting facilities that enhance the growth of tourism (Gao, Huang & Huang, 2008). Rural tourism is also associated with sustainable tourism (Bramwell, 1994), a platform that encourages the preservation of old aged culture and natural attractions. The very essence of rural tourism attracts visitors and tourists.

PNG as a tourist destination has a lot to offer in rural tourism (Imbal, 2010: Reggers et al., 2016: Ford et al., 2019). It has the potential to promote sustainable tourism (N'Drower, 2014). Within the umbrella of rural tourism, the following niche markets can be tapped into: community-based tourism (Wearing &McDonald, 2002), eco-tourism (Guaigu, 2014: Jones, 2005), cultural tourism (Whitford & Dunn, 2014), agri-tourism (Ammirato & Felicitti, 2014) and indigenous tourism (Zeppel, 2006: Hinch & Butler,1996: Taylor, 2017).

Methodology

Melanesian Research Framework (MRF) (N'Drower, 2020) was employed for the study where twelve tourism communities were invited to share their experiences as tourism entrepreneurs operating within their local village communities in rural PNG. Discussions of indigenous research methodology exist outside of tourism and offer insights into this aspect of tourism research. Meyer (2008) states that for current issues affecting indigenous people to be addressed effectively, new theories must be drawn from their ancient ways of knowing. It must be noted that one element that links diverse places and connects indigenous methodologies is the concept of colonisation; in many cases, these places have been dominated by Western approaches, which may not be appropriate for them. "Indigenous methodologies tend to approach cultural protocols, values and behaviours as an integral part of methodology" (Smith, 1999 p. 15).

Urion, Norton, and Porter (1995) argued that indigenous research methodologies should not be defined as this could prevent studies from being benchmarked inappropriately against European models. Story-telling was the instrument used in engaging with the participants. The Melanesian Research Framework is an indigenous research approach that integrates PNG's cultural values and practices.

Informal conversations in the form of storytelling were held for almost an hour. The shortest conversation took about 40 minutes while the longest was approximately an hour and 30 minutes. Connelly & Clandinin (1990) assert that storytelling allows people to express the different ways they experience the world. This can be particularly useful in indigenous research. Bishop (1999) argues that stories allow people to share meanings and unravel the truth, and stresses the importance of collaborative storytelling in indigenous research to help avoid researcher bias.

The importance of using a research instrument that is comfortable and convenient for indigenous people was also stressed by Hart (2010), who noted that storytelling is an everyday experience for indigenous people. The contrast between an embraced or developed method and spontaneous activity of people as in the form of storytelling is not as systematic or established. The storytelling can come from different conversational angles, thus allowing the listener to tie in neatly within the broader scope for one's relevance. If storytelling is a medium of expressing socio-political, cultural or economic insights, indigenous people have over time used that as a conduit nowadays for economic progress that befits the immediate communities. It is all the more relevant that the ideological position must always be for the good of the community and in that regard, social capital is expressively viable here.

The approach for the conversations was to make it as in-depth as possible with the help of four main theme questions, which guided the conversations: considerations when preparing for community-based tourism (CBT), structures and systems for operating a CBT operation, resourcing CBT and developing skills and local capacity to deliver CBT. With the help of key informants, the snowball technique was adopted to identify potential research participants. Storytelling being the main research instrument was used to engage with the participants in informal conversations. Participants were given the freedom to tell their stories at their own pace and the way they wanted to tell them. A voice recorder was used to capture the different stories shared by the participants. This was also considered an important process in ensuring trustworthiness and avoiding data misrepresentation. The recorded stories were then replayed and transcribed. The transcripts were analysed manually using thematic analysis.

Results

Data collected for this research is considered important in encouraging tourism in rural communities. Results obtained are presented in the form of direct quotes. This process is deemed appropriate as it embraces the purpose of the Melanesian Research Framework and that is to allow indigenous people to truly represent themselves by sharing their experiences and stories willingly without being encrypted, which could allow for the possibility of being misinterpreted. Shared in the following paragraphs are some conversations by the participants in light of the purpose of this paper.

The communities visited were friendly and sincere in sharing their experiences including the struggles they encountered, the excitement and motivations for embarking on tourism and

their hopes for better tourism set up in their respective communities. The highlight of the stories by the participants was the support they received from each other to build their tourism ideas as an economic activity into reality. They also shared that tourism is a community effort, for instance, one participant commented that "different clans are engaged to act as tour guides/porters and carriers, so in that way, the benefits of tourism trickle down to the community". Rural communities rely on family ties to sustain themselves in order to survive as a family/clan/tribe and community; not just for the sake of tourism but other activities such as bride price and compensation.

Pooling of resources by community members is a vital element of community survival and success. This was demonstrated by a community invited to participate in this study. "In 2015, we as a village decided to go into tourism, we built rooms for tourists and a conference room. We also dug the road and cleared the path that leads to the rooms." These communities are aware that tourism alone will not be their saving grace but yet they enjoy being part of tourism activities: "we like to show other people our culture and at the same time meet new people and learn new things". A topic raised during the conversations was the tourism challenges experienced by tourism resource owners. One participant commented, "I'd like to learn how to do online marketing so I do not have to rely on other people to market for me", while another commented, "the tour operators make more money than us even though we own the attraction sites and do cultural performances". The tour operators being referred to, are foreign-owned. It was stressed again that these communities continue to promote because they are related either through family ties or speak the same local vernacular. One commented, "the cultural group we formed consists of members from the two main clans". This quote is taken from one of few female participants, "It's good to help each other. My cousins helped me cut grass along the track, we help each other".

The cultural practice of collectivism by Papua New Guineans mirrors the comments of the participants stating that tourism is a community effort. (Davidsson & Honig, 2003) also share similar sentiments saying that cooperation by members of disadvantaged communities towards business start-ups is a form of social capital.

Discussion

Melanesian values and social capital

The value of maintaining relationships within clans and tribes in PNG is vital for survival. It is a form of social security and a foundation for social capital within rural communities, which means that understanding tourism development requires an understanding of the clan and tribal networks (Taylor, 2017: Hwang & Stewart, 2016).

The value of reciprocity (bekim, bekim bek) – the concept of reciprocity is closely linked to the value of clan and translates into an ethic of helping one another in times of need and returning the favour (Diedrich et al., 2019: Hwang & Stewart, 2016).

The value of food (*kaikai, mumu*) – the sharing of food is central to the PNG culture, being hospitable to visitors, families and friends as a token of friendship, appreciation and goodwill. The sharing of food is an important protocol for researchers seeking to establish rapport, negotiation, consultation and during the storytelling moments.

The value of ancestors (*tumbuna, tambaran*) – acknowledging ancestors guides how people within a village relate to each other. Either they could be descendants of the same ancestor or their ancestors are related. Having a common ancestor strengthens the relational ties amongst clan members, which contributes to working together and enforces the already established network, therefore manifesting as social capital.

The value of ritual (*taboo, singsing, lotu*) involves the preparation leading to an event or ceremony and sets out the actions that must be performed by those who initiate and those who participate in an event so the desired outcomes of the event will be achieved. The PNG diverse culture is a draw card for tourists, hence, the display of traditional songs and dances by villages for the sake of tourism is a community effort. Different members of a village join to either prepare the traditional attire (bilas) or participate as dancers.

The value of leadership (*hetman*) – an important element in rural communities is respecting and acknowledging the local leadership. Hence, an important promoter of effective tourism is good governance. Leadership portrayed by an individual who has the support of the community to venture into tourism is essential in promoting and sustaining tourism.

The value of education (*skul*) – in contemporary PNG, there is a belief that education can lead to better and improved ways of living. Tourism is an introduced concept, which requires tourism resource owners to learn the skills in managing their tourism activities.

The value of compensation (*peibek, bekim, birua*) – compensation refers to giving some form of payment to someone or a group of people after causing grievance to them. It aims to maintain peace so there are no tribal fights or ethnic clashes which usually end in bloody situations. Awareness of the importance of compensation would be critical to understanding patterns of interaction between communities involved in tourism. The value of work (*wok*) – refers to the importance of putting in effort and making sacrifices to achieve one's goal.

In rural PNG social capital is an important asset that facilitates community survival and livelihood. The value of kinship is deeply rooted in PNG's traditional culture. Thus, the concept of social capital has prompted communities to work together and promote tourism as a community initiative. In ensuring that there is a promotion of unity in the community, vital elements such as maintaining relationships, reciprocity among the people and embracing kinship patterns are considered irreducible indigenous essentialism (Scott, 2007).

Enhancing social relationships through tourism

Indigenous people may not have had a developed process that could be aligned to the rigorous cognitive development as is seen elsewhere (western societies). However, their way

of thinking and explanation was never conclusive but intrinsically relational. By way of coherence it would have seemed incongruent (had it not been for a process) and illogical. Implicitly, the trajectory that lies in the formulation of concepts and expression for an indigenous may not have consistency or rationality but there was a certain level of thinking involved as Kincheloe notes "There have to be modes of thinking that transcend the formal operational ability to formulate abstract conclusions, understand cause-effect relationships, and employ the traditional scientific method to explain reality" (Kincheloe, 2000, p.24). Such reality of the cause-effect relationship of indigenous people is implicit, meaning there is no systematic process of knowing (epistemological approach) as yet but it is not that there is no formal interrogation to accumulate knowledge.

Moreover, on the vantage point of being explicit, the reality (natural as well as supernatural) by which an indigenous begins to enquire in fact, draws from the array of information in existence to ascertain one's relationship. That type of relationship here extends beyond the formal taxonomy and does not necessarily portray the bond and trust that it tends to exemplify. Similarly, concepts such as affection, cognition and self in the classical epistemological development would still fall short in the indigenous context. At that juncture, how is it possible that tourism can still enhance the social relationship? We need not mix the economy of tourism with the social relationship but the fact remains obvious, despite its entrepreneurial linchpin, its social responsibility will continue to be maintained. Tourism exists for the sake of the people and they are the sole construct of the activity by which they benefit economically or in the words of Marti Siisiäinen it impacts the "sociological essence of communal vitality." Therefore, seeing under such current, it is all the more visible to note that economic benefit knits the mutual relationship among the people since it is a joint venture that they have initiated. At the outset, it is indeed verifiable that social relationship is a residue in the tourism industry. Maintaining a collegial approach and sustaining familial ties are expressive in the form of relationships.

Social capital in as much as social relationship threads on the same vein to maintain "moral obligations and norms, social values, (especially trust) and social networks (especially voluntary associations)" (Siisiainen, 2000). Relationships as in social capital must embrace the crux of enhancing social value especially trust because it will deteriorate when there is no trust established between the various individuals or community organizations. Trust must rid itself of bias and disparity should there be the sustainability of livelihood and the combination of capital assets like social, human, natural, physical and financial capital (Diedrich et al., 2019), which are basic elements that ought to be in existence should it be in economics, social structure, or politics.

The researched communities were optimistic about tourism and the potential for conflicting interests amongst community members arising from competition in CBT and other tourism-related activities were not reported in the present study, despite it being suggested as a problem by some of the early anthropological analyses (Halvaksz, 2006: Martin, 2008). In this study, it seemed that each of the tourism ventures, especially in the Highlands, was established by either an individual and/or his or her family unit. The development of tourism

did not, however, create competition or conflict, rather it triggered more enthusiasm from the collective social group to ensure success. For instance, if Betty's Lodge had tourists at her place, she would arrange with other tourism operators to benefit by showing their traditional dance or selling their artifacts so that the benefits through tourism are shared between the lodge's owner and others who are involved in tourism.

In Chimbu, the two main lodges that could bring in tourists without relying on tour operators were Betty's Lodge and Camp Jehovah Jireh. However, they relied on the other tourism providers within their community to facilitate the tourism activities for the tourists. In this scenario, social capital is being relied upon to enhance tourism within the area. It must be noted that the tourism providers including Betty's Lodge and Camp Jehovah Jireh are related through genealogy ties and speak the same mother tongue. The concept of involving everyone is driven not only by profit but also the importance of maintaining cultural ties and relationships. However, if, outside tour operators were involved, the purpose and outcome of tourism in this particular scenario might not be empowering for the locals. McCannell (1992, p. 28) commented that successful tourism is "profit without exploitation".

Having at least two major tourism outlets that were indigenous to the community that directly brought in tourists also minimised the impact of economic leakage within the community. This experience was quite different to the other communities visited who relied entirely on other stakeholders, mainly tour operators, who were seen as the major key players in tourism being operationalized in the communities. For any community to share a similar experience with that of Chimbu, at least one tourism operator from within the community must have the resources to source tourists directly. Tourism will then be embraced by the rest of the community, as social capital is already an existing asset, emphasizing that tourism is not only a profit-driven industry but also a social-cultural phenomenon. Oakes (2005), and Stronza (2001) commentated that locals are also part of the tourism encounter and in all fairness, have agency and control, however, when tourism is being dictated to them and there is an imbalance of power, locals become spectators in their own communities. A participant commented "the tour operators need us to make tourism happen in PNG, they don't own the tourism resources/sites/activities. But yet they make us feel like we need them to survive. We, the local tourism resource owners, are being cheated and exploited because we do not have the capacity and resources to market ourselves effectively." Hence, utilising social capital through CBT can lead to a more fruitful tourism encounter. Another person shared similar sentiments, "We should be able to market ourselves and get tourists without relying on tour operators. In our community, we plan to rebuild what was lost in the fire, build a proper hut that will host the selling of our arts and crafts and bilums." These were not just spoken words but words put into action. Eight months after visiting this particular community, I was sent a photograph of a newly built hut, which the women use as the designated space to sell their bilums and other arts and crafts when tourists visit (Figure 1), which was achieved through community cooperation.

Figure 1: Newly built hut for arts and crafts

Recommendation

For tourism to survive in rural communities in PNG, the local cultural system of the community must be understood such as the different clans that make up the community. The relationship within the clans and between the clans, the different roles each clan is responsible for within the community. The relationship between neighbouring communities is equally important because of the multiplier effect tourism has on society.

Tourism should be an initiative suggested from within the community and not imposed on the community by an outside source. This study has shown that social capital is a key element in sustaining tourism in rural communities. Hence, social capital is the collective pulling of resources by community members in achieving a common purpose. Social capital can only be drawn from within the community and not from an outsider. Therefore, it is recommended that tourism stakeholders allow rural communities to take leadership and ownership of their tourism affairs and not dictate tourism to them.

Conclusion

PNG's cultural practices have been an enabler of tourism, especially in the rural communities where in some remote areas basic infrastructure is still lacking to facilitate progress in both human and economic development. Cultural practices in the form of maintaining relational ties for thousands of years has indeed been the foundation of clans and tribes. Social capital in the form of a clan member helping another is a common inter-generational practice, which has also allowed individual members of clans, tribes and villages to survive. This concept is now being adopted to suit the needs of contemporary PNG as more rural dwellers venture into tourism within their local communities.

The communities visited were optimistic about tourism despite the low turnover of tourists. They are aware that their livelihood must not depend on tourism alone. But despite this, they see tourism as an avenue to share their culture with the rest of the world and to learn about other cultures and meet new people. Tourism not only allows them to meet the outside world

but also presents the opportunity for them to work together as a community, which in turn strengthens their relational ties and deepens the bond amongst clan/tribe/village/community members. This in turn empowers future tourism initiatives and other community projects. Figure 2 illustrates the connection between social capital and tourism. It reveals that social capital in PNG's context is embedded in genealogical ties.

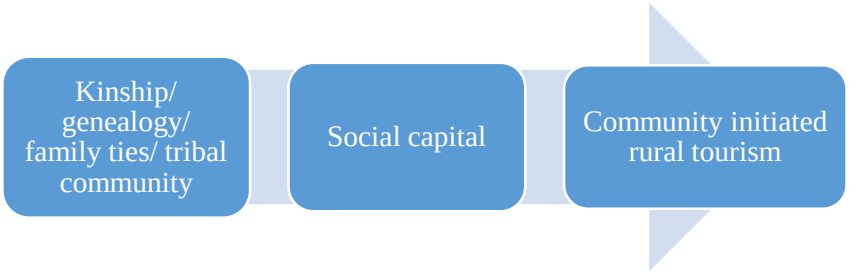


Figure 2: Social capital and rural tourism

References

Ammirato, S., & Felicetti, A.M. (2014). The agri-tourism as a means of sustainable development for rural communities: a research from the field. *International Journal Interdisciplinary Environmental Studies 8* (1), 17-29. DOI:10.18848/2329-1621/CGP/v08i01/53305

Bishop, R. (1999). Kaupapa Maori Research: An indigenous approach to creating knowledge. In N. Robertson (Ed.). *Maori and psychology: research and practice* – The proceedings of a symposium sponsored by the Maori and Psychology Research Unit. Hamilton: Maori & Psychology Research Unit. Available at https://researchcommons.waikato.ac.nz/bitstream/handle/10289/874/1999?sequence=1

Bourdieu, P. (2011). The forms of capital (1986). *Cultural theory: An anthology*, *1*, 81-93.

Bramwell, B. (1994). Rural tourism and sustainable rural tourism. *Journal of Sustainable Tourism*, *2*(1-2), 1-6.

Connelly, F. M., & Clandinin, D. J. (1990). Stories of experience and narrative inquiry. *Educational Researcher*, *19*(5), 2–14. DOI:10.3102/0013189X019005002

Davidsson, P., & Honig, B. (2003). The role of social and human capital among nascent entrepreneurs. *Journal of Business Venturing*, 18, 301–331.

Diedrich, A., Benham, C., Pandihau, L., & Sheaves, M. (2019). Social capital plays a central role in transitions to sport fishing tourism in small-scale fishing communities in Papua New Guinea. *Ambio, 48*(4), 385–396. DOI:10.1007/s13280018-1081-4

Douglas, N. (1997). The fearful and the fanciful: early tourists' perceptions of Western Melanesia. *Journal of Tourism Studies, 8*(1), 52–61.

Fitzpatrick, P. (2011). *Good reading and writing in Papua New Guinea*. Available at http://asopa.typepad.com/asopa people/2011/11/good-reading-and-writing-in-papua-new-guinea.html

Flora, J. L. (1998). Social capital and communities of place. *Rural Sociology*, 63(4), 481e506.

Ford, A., Carr, A., Mildwaters, N., & Summerhayes, G. (2019). *Promoting Cultural Heritage for Sustainable Tourism Development: Madang, Papua New Guinea*. https://DOI.org/APO-242516

Franklin, K. J. (2007). Framework for a Melanesian world view, *Catalyst*, *37*(1), 25–52.

Gao, S., Huang, S., & Huang, Y. (2009). Rural tourism development in China. *International Journal of Tourism Research*, *11*(5), 439-450.

Guaigu, G. (2014). Rainforests and rural village ecotourism venture in Papua New Guinea: A case study. In B. Prideaux (Ed.), *Rainforest tourism, conservation and management: Challenges for sustainable development* (pp. 213–225). Third Avenue: NY: Routledge.

Hart, M. A. (2010). Indigenous worldviews, knowledge, and research: The development of an Indigenous research paradigm. *Journal of Indigenous Social Development*, *1*(1A), 1–16. Available at https://scholarspace.manoa.hawaii.edu/handle/10125/15117

Halvaksz, J. A. (2006). Becoming local tourists: Travel, landscapes and identity in Papua New Guinea. *Tourist Studies*, *6*(2), 99–117. https://DOI.org/10.1177/1468797606071471

Hinch, T., & Butler, R. (1996). Indigenous tourism: a common ground for discussion. *Tourism and indigenous peoples.*, 3-19.

Hwang, D., & Stewart, W. P. (2017). Social capital and collective action in rural tourism. *Journal of Travel Research, 56*(1), 81-93. DOI: 0.1177/0047287515625128

Imbal, J. (2010). Contemporary challenges facing the development and management of culture tourism in Papua New Guinea. *Contemporary PNG Studies*, *12,* 12-28.

Jones, S. (2005). Community-based ecotourism: The significance of social capital. *Annals of Tourism Research,* 32(2), 303-324. DOI: 10.1016/j.annals.2004.06.007

Kincheloe, J. L. (2000). Making critical thinking critical. Perspectives in critical thinking: Essays by teachers in theory and practice. *Counterpoints*, *110*, 23-40. Available at https://www.jstor.org/stable/pdf/42975935.pdf

Lane, B. (1994). What is rural tourism? *Journal of Sustainable Tourism*, 2(1–2), 7–21. DOI:10.1080/09669589409510680

Martin, K. (2008). The work of tourism and the fight for a new economy: The case of the Papua New Guinea mask festival. *Tourism, Culture and Communication, 8*(2), 97–107. DOI: 10.3727/109830408785070749

MacCannell, D. (1992). *Empty meeting grounds: The tourist papers*. London, England: Routledge. DOI:10.4324/9780203412145

Meyer, M. (2008). Indigenous and authentic. Hawaiian epistemology and the triangulation of meaning. In N.K Denzin, Y.S Lincoln & L. T Smith (Eds.), *Handbook of critical and Indigenous methodologies (*pp. 217–231). Thousand Oaks, CA: SAGE.

Nahapiet, J., & Ghoshal, S. (1998). Social capital, intellectual capital, and the organization advantage. *Academy of Management Review*, 23(2), 242–266.

Nair, V., Munikrishnan, U. T., Rajaratnam, S. D., & King, N. (2015). Redefining rural tourism in Malaysia: A conceptual perspective. *Asia Pacific Journal of Tourism Research*, *20*(3), 314-33

N'Drower, F. P. (2014). Sustainable rural tourism practices in Madang province, Papua New Guinea. *Contemporary PNG Studies, 20,* 90-102.

N'Drower, F.P. (2020) *Developing an Indigenous research tool to explore the effectiveness of community-based tourism in rural Papua New Guinea.* PhD thesis, James Cook University. DOI: http://dx.doi.org/10.25903/pygj-2v81

Oakes, T. (2005). Tourism and the modern subject: Placing the encounter between tourist and other. In C. Cartier & A. A. Lew (Eds.), *Seductions of place: Geographical Perspectives on Globalization and Touristic Landscape* (pp. 36–55). Abingdon, United Kingdom: Routledge.

Pretty, J., & Ward, H. (2001). Social capital and the environment. *World Development, 29*(2), 209-227.

Reggers, A., Grabowski, S., Wearing, S. L., Chatterton, P., & Schweinsberg, S. (2016). Exploring outcomes of community-based tourism on the Kokoda Track, Papua New Guinea: a longitudinal study of Participatory Rural Appraisal techniques. *Journal of Sustainable Tourism, 24*(8-9), 1139-1155.

Richardson, J. (1986). *Handbook of Theory and Research for the Sociology of Education*. Westport, CT: Greenwood

Saarinen, J., & Lenao, M. (2014). Integrating tourism to rural development and planning in the developing world. *Development Southern Africa, 31*(3), 363-372.

Scott, M. W. (2007). Neither 'new Melanesian history' nor 'new Melanesian ethnography': Recovering emplaced matrilineages in Southeast Solomon Islands. *Oceania, 77*(3), 337-354. Available at https://www.jstor.org/stable/pdf/40495563.pdf

Sharpley, R., & Sharpley, J. (1997). *Rural tourism. An introduction* (pp. 1-165). International Thomson Business Press.

Siisiainen, M. (2003). Two concepts of social capital: Bourdieu vs. Putnam. *International Journal of Contemporary Sociology,* 40(2), 183-204.

Smith, L. T. (1999). *Decolonizing methodologies: Research and Indigenous peoples.* Dunedin, New Zealand: Zed Books.

Stronza, A. (2001). Anthropology of tourism: Forging new ground for ecotourism and other alternatives. *Annual Review of Anthropology, 30*(1), 261–283. DOI:10.1146/annurev.anthro.30.1.261

Taylor, S. R. (2017). Issues in measuring success in community-based Indigenous tourism: elites, kin groups, social capital, gender dynamics and income flows. *Journal of Sustainable Tourism, 25*(3), 433-449.

Urion, C., Norton, R., & Porter, T. (1995). Roundtable Discussion #1. *Canadian Journal of Native Education*.

Wearing, S., & McDonald, M. (2002). The development of community-based tourism: Re-thinking the relationship between tour operators and development agents as intermediaries in rural and isolated area communities. *Journal of Sustainable Tourism,* 10(3), 191-–206. DOI:10.1080/09669580208667162

Whitford, M., & Dunn, A. (2014). Papua New Guinea's Indigenous Cultural Festivals: Cultural Tragedy or Triumph?. *Event Management*, *18*(3), 265-283.

Zeppel, H. (2006). *Indigenous ecotourism: Sustainable development and management*. Oxfordshire, United Kingdom: CABI Pub. DOI:10.1079/9781845931247.0000

Zhao, W., Ritchie, J. B., & Echtner, C. M. (2011). Social capital and tourism entrepreneurship. *Annals of Tourism Research, 38*(4), 1570-1593. Retrieved from https://DOI.org/10.1016/j.annals.2011.02.006

Acknowledgement

I would like to thank Professor Peter K. Anderson and Dr Martin Daniel for reviewing this paper. However, responsibility for any errors of fact or opinion remains with the author.

Authors

Harry Gahare (late) was a senior tutor within the School of Humanities and is attached with the School of Humanities in the University of Goroka, Papua New Guinea. Email: harrygahare@gmail.com

Fiona N'Drower (PhD) is a lecturer in the Department of Tourism & Hospitality Management, Faculty of Business and Informatics at Divine Word University. Email: fndrower@dwu.ac.pg or fndrower@yahoo.com

Essential pillars of e-government for developing countries

Martin Daniel

Abstract

The growth of affordable digital technologies is moving governments from paper-based workflow to electronic processes, leading to the development of e-government. These changes are made possible as governments progress to address the essential pillars of e-government. This paper discusses the essential pillars, around which e-government is developed, including citizen centricity, standardised infrastructures, back-office reorganisation, governance, new organisational models, social inclusion, people, processes, technology and resources. The paper also provides some recommendations for developing countries so that these pillars are considered to shift from paper-based procedures to electronic processes, in the development of e-government.

Keywords: back-office reorganisation, citizen needs, citizen-centric approach, citizen-centricity, developing countries, electronic government (e-government), electronic processes, financial resources, governance, human resources, interoperability, new organisational models, paper-based procedures, people, pillars of e-government, processes, service-oriented approach, social inclusion, standardised infrastructures, technology, virtual organisation.

Introduction

"The rise of the digital society, internet and affordable computing have brought about a shift from traditional government offices with paper-based processes towards electronic resources such as the web and intranets" (Kaczorowski, 2014, para. 1). This shift has affected governments and their citizens, businesses and other stakeholders.

E-government is not just about technology but how to develop a civil society (Kaczorowski, 2014). It is not just about systems and specifications but government; computers and websites but citizens and stakeholders; not just about translating processes but transforming processes (Boughzala, Janssen, & Assar, 2015; Satyanarayana, 2006). They create a challenge for governments to use technology to increase service quality, which includes reducing costs and improving connectedness with citizens and others.

E-government success depends on various pillars (Figure 10), including "citizen centricity, standardised common infrastructures, back-office reorganisation, governance, new organisational models and social inclusion" (Kaczorowski, 2014, para. 3), as well as people, processes, technology and resources (Kalam, 2006). These pillars are essential themes around which e-government is developed.

This paper will discuss these pillars, which need to be considered holistically in the planning and implementation stages of e-government to obtain the desired outcomes (Satyanarayana, 2006). The paper will also present some recommendations that could be used to address these pillars for e-government success in developing countries, as identified in the literature.

Figure 10: Pillars of e-government for developing countries.

Citizen centricity

Citizen-centricity is about shifting the focus from government operational requirements to delivering services based on citizens' needs. It places the citizens at the centre of public service delivery (Kaczorowski, 2014). "Citizens today expect more transparent, accessible and responsive services" (Dudley, Lin, Mancini, & Ng, 2015, para. 3). They desire efficient and easy-to-use services that meet their needs (Berntzen, 2013). The citizen-centric approach will not only meet citizen expectations but also enable them to participate and make decisions in the government processes (United Nations, 2010). This approach deals with complicated bureaucracy and unnecessary silos in the government systems, which are usually rigid and inflexible (Lesser, 2018). A citizen-centric approach can enable governments to "achieve essential efficiency gains and improve service delivery levels, increase usage of online services, thereby improve sustainability and encourage investment in e-governance [and] improve citizen satisfaction with government services" (Malik, Gupta, & Dhillon, 2014, p. 92).

Standardisation

"Standardisation is the process by which specifications are set ... [to] ensure that devices, systems and services retain the ability to connect and interoperate with each other, boosting innovation, [openness] and [competitiveness]" (European Commission, 2019, para. 1). It has many benefits but ultimately maintains consistency throughout the organisation, minimises overall cost (IP Partners, 2019), ensures "scalable infrastructure, easier and faster deployment, streamlined IT management, efficient communications and resolution of issues, and simplified and collaborative decision making" (HTL, 2017). Standardisation is required to maximise interoperability with the use of technology (European Commission, 2019; Kaczorowski, 2014). "Given the complexity of government structures and processes, which have evolved with different, poorly coordinated legacy systems, few governments can afford

to take the steps taken by the private sector towards a consistent standardisation of ICT" (Kaczorowski, 2014, para. 6). However, many countries are developing their own national e-government strategies to achieve interoperability between their systems, processes, software and networks.

Back office reorganisation

Back-office refers to the part of an organisation that is not client-facing and supports the front office (Indeed, 2020). It is usually responsible for accounting and finance, information technology, data management and human resources (Indeed, 2020; Verint, 2020). The back office is sometimes referred to as operations as employees in the back-office usually perform the organisation's operations while the front office interacts with and serves the clients. Reorganising the back office, which is closely related to standardisation, involves "automating ... routine administrative processes, freeing staff to focus more on the delivery of services". "Governments achieve significant cost reduction [and improved service delivery] when they reorganise their back-office processes before bringing services online" (Kaczorowski, 2014, para. 7). Reorganising the back office helps to realign the government's processes to support the front office in serving the citizens based on their needs.

Governance

Governance refers to structures, systems, practices and processes that are designed to "define how decisions are to be made and establish the organization's strategic direction; oversee the delivery of its services; the implementation of its policies, plans, programs, and projects; and the monitoring and mitigation of its key risks; and report on its performance in achieving intended results and use performance information to drive ongoing improvements and corrective actions" (Canadian Audit & Accountability Foundation, 2010, para. 2 - 4). A proper governance structure enables careful planning and coordination of whole-of-government initiatives. In governments, ministries and agencies were usually responsible for driving their strategies, leading to "a lack of coordination and interoperable systems, and duplication of solutions" (Kaczorowski, 2014, para. 8). This resulted in a waste of unnecessary resources and efforts, leading to unfulfilled service needs.

New organisational models

"An organizational model describes the objectives and the structure of an organization in terms of roles, norms, relations between roles and interactions between roles" (IGI Global, 2020, para. 1). The current traditional organisational model where organisations operate in isolation will not be able to meet the current citizen demands on time. This necessitates the need to have new organisational models, which require the creation of network virtual organisations. A virtual organization refers to a network of agencies, joining together "to provide innovative, high-quality products or services instantaneously in response to customer demands" (Advameg, 2020, para. 1). A virtual organisation has more resources and capabilities than each organisation alone as they mobilise their resources and capabilities to meet citizen needs.

Social inclusion

Social inclusion can be defined "as the process of improving the terms of participation in society, particularly for people who are disadvantaged, through enhancing opportunities, access to resources, voice and respect for rights" (United Nations, 2016, p. 17). These disadvantaged individuals and groups include those with a disability or who are located in remote areas. Governments have realised that the expansion in ICT can have the potential to improve social inclusion, by reducing the gap between the digital haves and have-nots (Kaczorowski, 2014).

People

People, both in and outside, government agencies play a vital role to ensure the success of e-government initiatives (Abdalla, 2012; Kalam, 2006; Moatshe, 2014). E-government can bring transformation to agencies and therefore requires substantial resources including financial and human resources with the required knowledge, skills, experiences and commitment (Abdalla, 2012; Moatshe, 2014; Yadav & Singh, 2012). Staff may require appropriate training for a new set of skills. Senior officers and their staff need accurate information about e-government to secure their support and contribution. Active citizen participation also contributes to the success of e-government, focusing on services that meet citizen needs and expectations (Moatshe, 2014).

Processes

This pillar is closely related to back-office reorganisation. As previously mentioned, e-government is not about process automation with its inefficiencies, but process transformation and creation of relationships between the government and its citizens, business and stakeholders (Abdalla, 2012; Kalam, 2006). Government processes are usually slow, inflexible and operate in isolation, and usually lack service-centricity and citizen-focus. These issues lead to long queues and unnecessary intermediaries at service delivery points. "Typically, citizens make multiple visits to government offices unsure of the outcome or quality of service, mystified by government procedures and at the mercy of government officials" (Kalam, 2006, p. 31). Transforming the processes can have the potential to eliminate these inefficiencies.

Technology

Although e-government is not just about technology, the latter is an enabler and can have the potential to transform government processes, leading to achieving e-government outcomes (Abdalla, 2012; Moatshe, 2014; Reddick, 2018). E-government success depends on having an overall architecture, strategy and roadmap, standardised infrastructure including well-established communication networks, and adopting a service-oriented approach to development (Abdalla, 2012; Kalam, 2006). These issues can be carefully addressed by having a proper governance structure for e-government implementation. The structure should specify e-government principles, policies, architecture, infrastructure, applications, investment and prioritisation (Kalam, 2006).

Resources

As previously mentioned, e-government requires financial and human resources, and a huge investment in technology (Abdalla, 2012; Moatshe, 2014). A lack of resource commitment and sustainability can lead to e-government failure or delays. Adequate funding is necessary for the lifetime of e-government initiatives and needs consideration at the beginning of projects. This requires political and top management commitment and support, and proper planning and coordination so that resources are utilised in a coordinated way for maximum e-government benefits. Further, governments could consider public-private partnership arrangements to mobile resources from the public and private sectors (Kalam, 2006; Yadav & Singh, 2012). In this way, the government provides funding while the private sector provides technical expertise and management efficiency to embark on complex e-government projects.

Discussion

Although e-government presents various challenges, many countries with differing levels of e-government development indicate that governments that are ambitious, visionary and committed can overcome these challenges. Such governments are able to re-engineer their infrastructures and processes and create new ways to enable two-way interactions with citizens, businesses and stakeholders. They are able to gain the benefits of e-government and achieve a more effective government overall. It is, therefore, essential to consider these pillars of e-government as identified in the literature.

Having a citizen-centric approach requires that governments "develop the capacity to act as a single enterprise so that citizens feel that they are being served by one organisation. [They need to] organise themselves around citizen demands and expectations [and] develop flexible organisational structures" (Kaczorowski, 2014, para. 4). Governments need to change focus from prioritising their operations to prioritising the service needs of the citizens. In this way, the governments would be able to redesign their processes around meeting the needs of the citizens in a cost-effective and timely manner rather than simply automating their inefficient processes.

Standardisation is necessary to achieve full integration between agencies. Developing countries need to establish standards and national e-government strategies to achieve interoperability between their systems and processes. This would enable information and resource sharing to serve citizen needs (Kaczorowski, 2014).

Reorganising the back offices can help to manage e-government challenges and opportunities. This reorganisation leads to cost reduction, increased productivity and flexibility, simple organisational structures, greater interoperability and improve working conditions. It can also reduce citizen visits to public offices, quicker and improved service accessibility, greater transparency and ease of service usage (Millard & Iversen, 2004).

Appropriate governance structures are required at the national level to drive e-government strategies. This would ensure proper planning and coordination of e-government initiatives.

Appropriate policy frameworks are also necessary to support e-government development (Shivakumar, 2002; Talip & Narayan, 2011).

Since a new organisational model, virtual organisation, requires "joining up multiple organisations to achieve results that a single organisation could not achieve alone, this approach involves breaking down traditional structures based on separate functions and working flexibly and innovatively across boundaries to deliver better value to the citizens" (Kaczorowski, 2014, para. 9). Agencies should now work in partnership with other agencies and the public sector to embark on e-government initiatives, thereby, increasing efficiency in delivering and accessing services.

Effective e-government strategies address and achieve social inclusion by "improving the ability, opportunity, and dignity of those disadvantaged on the basis of their identity" (World Bank, 2020, para. 1). Social inclusion enables the disadvantaged to access services through effective e-government.

Government process reengineering involves radical thinking and redesigning processes to achieve significant improvements in performance including cost, quality, service and speed (Bhaskar & Singh, 2014). It "requires that an agency implement substantive reform in organizational structure, initiate a change in culture and mindset, train and improve skills of its people, and put in place appropriate supporting ICT infrastructure to enable online processes that are timely and efficient to both the user and the government agency" (Kalam, 2006, pp. 31-32).

E-government success depends on sustained resources. Proper planning and management would ensure limited resources are used effectively for purpose. Qualified personnel with relevant skillsets in both technical and non-technical areas need to actively participate in e-government development. Current staff may require training to upgrade their skills. Active citizen participation would ensure that processes are designed to their service needs.

Governments need to have an overall governance structure, strategy and roadmap, and a standardised infrastructure as previously mentioned (Abdalla, 2012; Kalam, 2006). Without these, e-government would be likely to fail in achieving its desired outcomes.

These e-government pillars are interrelated and should not be considered in isolation. Otherwise, those being unconsidered will affect e-government progress. For example, e-government development would be slow or delayed if limited resources are mismanaged or qualified personnel are not involved in the process.

Conclusion

The growth of the Internet and affordable technologies has moved governments from traditional offices with paper-based procedures to electronic processes to provide services that satisfy citizen demands. This has led to e-government development in various countries.

E-government is about using technology to transform government processes to increase quality in public services, which includes reducing costs and improving connectedness with citizens, businesses and stakeholders.

E-government success depends on including citizen-centricity, standardised common infrastructures, back-office reorganisation, governance, new organisational models, social inclusion, people, processes, technology and resources. These pillars are essential themes around which e-government is developed and need to be considered holistically in the planning and implementation process to obtain the desired e-government outcomes.

This paper discussed the essential pillars of e-government as identified in the literature. It also provided some recommendations that could be used to address these pillars so that e-government success is accomplished with the effective management of limited resources.

References

Abdalla, S. (2012). *An e-government adoption framework for developing countries: A case study from Sudan.* (Unpublished PhD thesis), Cranfield University. Retrieved from https://dspace.lib.cranfield.ac.uk/handle/1826/7314

Advameg. (2020). *Virtual organisations.* Retrieved 10 November 2021, from https://www.referenceforbusiness.com/management/Tr-Z/Virtual-Organizations.html

Berntzen, L. (2013). *Citizen-centric e-government services: Use of indicators to measure degree of user involvement in e-government service development.* Retrieved 10 November 2021, from https://pdfs.semanticscholar.org/6106/9f96af8c4fca88f3c07de25dd15a61c8735c.pdf

Bhaskar, H. L., & Singh, R. P. (2014). Business process reengineering: A recent review. *Global Journal of Business Management, 8*(2), 24-51.

Boughzala, I., Janssen, M., & Assar, S. (2015). E-government 2.0: Back to reality, a 2.0 application to vet *Case Studies in e-Government 2.0.* Retrieved 10 November 2021, from https://link.springer.com/chapter/10.1007/978-3-319-08081-9_1

Canadian Audit & Accountability Foundation. (2010). *What is governance?* Retrieved 10 November 2021, from https://www.caaf-fcar.ca/en/oversight-concepts-and-context/what-is-oversight-and-how-does-it-relate-to-governance/what-is-governance

Dudley, E., Lin, D.-Y., Mancini, M., & Ng, J. (2015). *Implementing a citizen-centric approach to delivering government services.* Retrieved 9 November 2021, from https://www.mckinsey.com/industries/public-and-social-sector/our-insights/implementing-a-citizen-centric-approach-to-delivering-government-services

European Commission. (2019). *ICT and standardisation.* Retrieved 12 November 2021, from https://ec.europa.eu/digital-single-market/en/policies/ict-and-standardisation

HTL. (2017). *Six benefits of IT infrastructure standardisation in the enterprise.* Retrieved 9 November 2021 from https://www.htl.london/blog/6-benefits-of-it-infrastructure-standardisation

IGI Global. (2020). *What is organisational model?* Retrieved 12 November 2021, from https://www.igi-global.com/dictionary/autonomous-agents-adopting-organizational-rules/21522

Indeed. (2020). *Back office: Definition and importance.* Retrieved 9 November 2021, from https://www.indeed.com/career-advice/career-development/what-is-back-office

IP Partners. (2019). *Standardising your IT infrastructure.* Retrieved 11 November 2021, from https://www.ippartners.com.au/it-news/benefits-of-standardising-your-it-infrastructure/

Kaczorowski, W. (2014). *The six pillars of e-government in the rise of the digital society.* Retrieved 10 November 2021, from https://www.politico.eu/article/the-six-pillars-of-e-government-in-the-rise-of-the-digital-society/

Kalam, A. P. J. A. (2006). *Introduction to governance and e-governance.* Retrieved 9 November 2021, from https://shodhganga.inflibnet.ac.in/bitstream/10603/19278/6/06_chapter%201.pdf

Lesser, R. (2018). W*hy it's time for governments to deliver citizen-centric reform.* Retrieved 9 November 2021, from https://www.centreforpublicimpact.org/now-time-governments-deliver-citizen-centric-reform/

Malik, P., Gupta, V., & Dhillon, P. (2014). Citizen-centric approach for e-governance: Looking at the service delivery through the eyes of the citizens. *International Journal of Applied Engineering and Technology, 4*(2), 91-100.

Millard, J. (2004, August). Reorganisation of government back-offices for better electronic public services. In *International Conference on Electronic Government* (pp. 363-370). Springer, Berlin, Heidelberg.

Moatshe, R. M. (2014). *E-government implementation and adoption: The case study of Botswana government.* (Doctor of Philosophy Doctoral thesis), University of Derby. Retrieved 10 November 2021, from https://derby.openrepository.com/handle/10545/337831

Reddick, C. G. (2018). Forward. In K. J. Bwalya (Ed.), *The e-government development discourse: Analysing contemporary and future growth prospects in developing and emerging economies*. Cape Town: AOSIS.

Satyanarayana, J. (2006). *E-government principles.* Retrieved 10 November 2021, from http://siteresources.worldbank.org/INTEGOVERNMENT/Resources/eGovPrinciplesJS5Nov06.ppt

Shivakumar, K. (2002). An overview of e-government. *International Symposium on Learning Management and Technology Development in the Information and Internet Age* Retrieved 9 November 2021, from https://pdfs.semanticscholar.org/5cb2/b8d2894d8b4f2c66edea8be9f7627f5c6d3f.pdf

Talip, B. A., & Narayan, B. (2011). *A proposed model for successful implementation of e-government in Malaysia.* Paper presented at the 7th International Conference on Information Technology in Asia, United States of America.

United Nations. (2010). *Improving e-government through a citizen-centric approach.* Retrieved 8 November 2021, from https://www.un.org/development/desa/newsletter/feature/2010/12/580.html

United Nations. (2016). *Leaving no one behind: The imperative of inclusive development*. Retrieved 11 November 2021, from https://www.un.org/esa/socdev/rwss/2016/full-report.pdf.

Verint. (2020). *The definitive guide to back office operations.* Retrieved 9 November 2021, from https://www.verint.com/engagement/resources/guide-to-back-office-operations/

World Bank. (2020). *Social inclusion.* Retrieved 10 November 2021, from https://www.worldbank.org/en/topic/social-inclusion

Yadav, N., & Singh, V. B. (2012). E-governance: Past, present and future in India. *International Journal of Computer Applications, 53*(7), 36-48.

Contribution

This paper contributes to the literature by organising and presenting e-government factors as essential pillars (themes) around which e-government is developed. In doing so, the paper shows the importance of ensuring that these pillars are considered in the planning, implementation and monitoring of e-government development.

Acknowledgement

I would like to dedicate this paper to late Professor Peter K. Anderson for his never-ending support in my PhD journey and for being the greatest mentor in my life. I will always remember him. I would like to thank him for reviewing this paper. However, responsibility for any errors of fact or opinion remains with the author.

Author

Martin Daniel (PhD) is an Associate Professor and the Dean of the Faculty of Business & Informatics at the Divine Word University in PNG. He holds a Bachelor's degree in Information Technology from the University of Queensland in Australia, a Master's degree in Information Technology from the Queensland University of Technology in Australia and a Doctor of Philosophy from the Divine Word University in PNG. His interests include researching and modelling technological developments (e.g. e-government) in PNG. Email: mdaniel@dwu.ac.pg and mdmartindaniel@gmail.com.

Human Development Index: PNG progress optimization model

Michaelynne Yaguro
Raunu Gebo Sarsoruo

Abstract

Human Development Index (HDI) is a holistic measure of human well-being, which is essential for a nation's strategic development plan. This paper aims to identify Papua New Guinea's (PNG's) government planning policies that improve the indicators of the PNG HDI and propose modifications to the indices to model the achievement of the PNG Vision 2050 goals. Modifying the indices would help refine policies that could enhance PNG to be ranked among the top 50 countries in the United Nations Development Program's Report. Currently, PNG HDI is ranked at 0.555 and is categorized as a medium human development country. Using an optimization model on current PNG HDI data through an objective function, a maximized ranking for PNG is generated. The objective function is solved through linear programming with a maximized value between 0.7 to 0.9, thus placing PNG amongst the top high human development countries and thereby achieving its' 2050 goal.

Keywords: Human development index, education index, health index, income index, optimization model, linear programming, data envelopment analysis.

Introduction

Every year, the United Nations Development Program (UNDP) publishes a report based on a country's Human Development Index (HDI) ranking. The HDI rank is simply a composite measure between 0 and 1 quantifying human well-being in terms of a country's education, health and income indices per annum.

Mariano et al. (2021) showed researchers to employ the use of the Data Envelopment Analysis (DEA) approach to further explore the human development index. It is a technique based on linear programming for determining the efficiency of decision-making in transforming a set of inputs into a set of outputs (Mariano et al,2021). Yekta et. al. (2018) describes DEA as a powerful mathematical programming method that measures the relative performance of a country in terms of human development and is defined based on the data given in the Human Development Reports that is published annually by UNDP (Despotis, 2001). A simplified index maximizing linear programming model is used to estimate an ideal value of the composite index for PNG with reference to neighbouring countries in the high human development group namely, Hong Kong, Singapore and Australia. The present values of the indicators (Table 2) giving the level of education attainment (EI), long years of living healthily (HI) and standard of living (II) reflects the current governing policies on a national scale. These values can be increased in the next 30 years (2020-2050) in accordance with the PNG Vision 2050 seven strategic focus areas. To minimize the high illiteracy and poverty rates, the nation focuses all its efforts on increasing the education index by promotion of

human capital development, gender, youth and people empowerment. In doing so, it strives to achieve the key outcome of changing and rehabilitating the mindset of all citizens (PNG Vision 2050). Even so, one of the economic growth projections involves increased downstream processing. The earnings from this will contribute to increasing the income index, where the economy is proposed to grow in real terms at an average of 4.5 per cent per year up to 2020 (PNG Vision 2050) increasing income per capita. Unfortunately, this is not true by computations according to GNI per capita data from Table 1. Hence, this paper analyses the current data available for PNG HDI through an ideal optimization model, which could be used to improve government policies that would improve each HDI indicator. The main constraints from the factors affecting the PNG HDI are illiteracy and poverty rates, challenging geographical terrains for important government services, Covid-19 restrictions and high infrastructure costs.

The results obtained from the DEA approach optimization model is important for PNG government planners to keep track of its 50 years' strategic development plan. A planner can check if the goals are achieved or not within a minimum of 5-10 years so changes can be made accordingly until the 2050 goals are achieved.

Literature review

Engineer et.al (2008) proposed the HDI as a criterion for optimal planning. The best optimal solutions obtained from the linear model that represents the composite indices of a planners' problem is the way forward for better and improved government policies for a nation's development. Such a model is necessary for implementation in PNG, where growth and development are paramount. A critical issue pointed out when estimating the HDI is that equal weights are assumed for its three component indices (Despotis, 2005). To address this issue, the DEA approach was used to assess the relative performance of a country's human development based on the data given in the Human Development Report annually by the UNDP. Mariano et.al (2021) further outlined the advantages, disadvantages and possibilities of applying different approaches to DEA in human development indicators. Hence, a simplified index optimization model is proposed for PNG in this paper. The PNG government has documented a 40-years strategic development plan through its National Executive Council which was documented and launched in 2009 (Papua New Guinea Vision 2050). A thorough analysis was done to check whether the PNG government was able to reach some of its strategic goals for 10-years up to 2020 as outlined in the PNG Vision 2050 booklet. The strategic focus areas that underpin the vision 2050 includes human capital development and wealth creation apart from the other five pillars. Current government policies need updating specifically on human capital development and wealth creation for increment in the current PNG HDI rank to achieve the vision "to be among the top 50 countries in the United Nations Human Development Index" (PNG Vision 2050, pp.13-14).

Current practices, challenges and problems

The PNG Vision 2050 strategic plan is divided into a 20-year plan called the PNG Development Strategic Plan 2010-2030 (PNG DSP 2010-2030) whose goal is for the country

to attain the middle-income status by 2030. The PNG DSP 2010-2030 mentions using the PNG Government's computerized general equilibrium model (PNGGEM) where appropriate to quantify the goals and scenarios. The problematic issue with this method is that equilibrium models are based on economics and have no correspondence to formal mathematical model concepts.

As discussed by Ambang (2019), the main challenge is measuring the progress of the strategies in place, which has an impact on the country as a whole. The country's ranking has been moving at snail-pace towards the overall goal as shown in Figure 1 below by the green line. This poses the question of whether or not PNG will achieve its set goals by 2050. The problem lies with authorities not being able to track the progress of planned development and policies that governs proper development. The sooner this is addressed, the higher the chances of acquiring the set goals. This paper shows a mathematical proven outcome of areas needing improvements if we are to reach the goal within the set time frame. The current HDI is 0.555 and ranks 155 out of 189 countries sitting in the medium human development group (UNDP 2020b). To reach the goal of ranking in the top 50s, the HDI has to be equal to or greater than 0.7 to qualify for high human development (UNDP HDI Report, 2020).

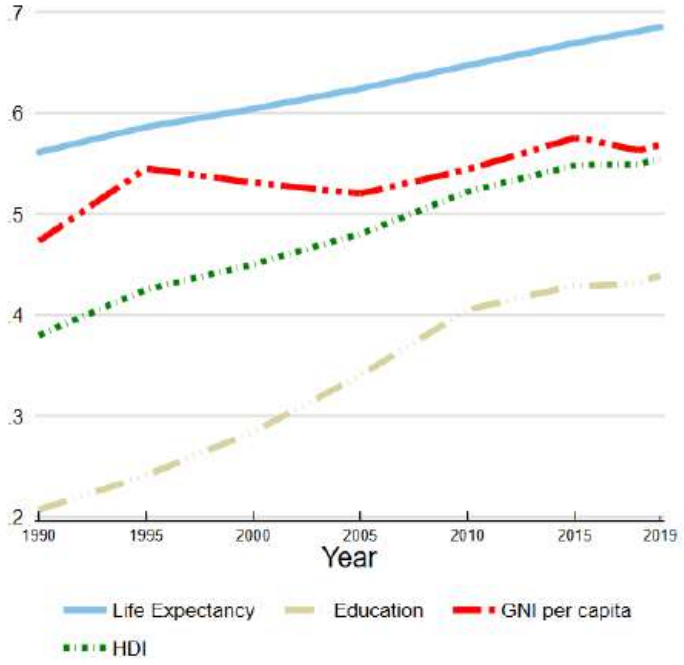


Figure 1: Trend's in Papua New Guinea's HDI component indices 1990-2019.
Source: UNDP HDI Report (2020)

Method

The first step involves computation and tabulation of current PNG HDI data in its raw form extracted from the UNDP PNG Human Development Report 2020. The raw data is then tabulated into its normalized form using the rules of HDI computation as stated by Gebo & Anderson (2019). The HDI, a composite statistic, that measures human well-being using three indices are calculated as follow;

1. Life Expectancy Index (LEI) $= \frac{LE-20}{85-20}$,

where LEI is 1 when Life Expectancy at birth is 85 and 0 when Life Expectancy at birth is 20,

2. Education Index (EI) $= \frac{MYSI+EYSI}{2}$,
   where;
   i) Mean Years of Schooling Index (MYSI) $= \frac{MYSI}{15}$ given that fifteen is the projected maximum of this indicator for 2025
   ii) Expected Years of Schooling Index (EYSI) $= \frac{EYS}{18}$ given that eighteen is equivalent to attaining a master's degree in most countries,
3. Income Index (II) $= \frac{\ln(GNIpc)-\ln(100)}{\ln(75{,}000)-\ln(100)}$,
   where II is 1 when GNI per capita is $75,000 and 0 when GNI per capita is $100.
4. The geometric mean of the three indices above is the calculated HDI, thus $HDI = \sqrt[3]{LEI.EDI.II}$.

In the second step, a model is developed as a government planner's problem with concepts derived from the DEA approach by (Despotis,2005) and a static closed economy model (Engineer et.al, 2008). In the next step, the planner's problem is solved through linear programming followed by results and discussions. Finally, a conclusion about the results is made.

HDI data normalization

HDI indicators with raw data extracted from the UNDP PNG Report 2020 is shown in table 1 for the years 1990-2019.

Table 1: Papua New Guinea's HDI trends based on consistent time series data and new goalposts. Source: UNDP Report (2020)

	Life expectancy at birth	Expected years of schooling	Mean years of schooling	GNI per capita (2017 PPP$)	HDI value
1990	56.5	4.7	2.3	2,289	0.380
1995	58.1	5.2	2.9	3,682	0.425
2000	59.3	6.3	3.3	3,368	0.450
2005	60.5	8.0	3.6	3,126	0.480
2010	62.0	9.8	4.0	3,661	0.522
2015	63.5	10.2	4.3	4,512	0.548
2016	63.7	9.9	4.6	4,414	0.549
2017	64.0	10.0	4.6	4,267	0.549
2018	64.3	10.0	4.6	4,152	0.549
2019	64.5	10.2	4.7	4,301	0.555

Observe from Table 1 that life expectancy at birth for Papua New Guineans has an increment value of 3 after every decade from 1990-2020. Expected years of schooling increased slowly from 4 to 10 by an increment of only 6 years within 3 decades with mean years of schooling currently at 4.7. Income earned by the country each year since 1990 grew at a slow pace with an increment of only $2000 from $2,000 to $4000**.** PNG was not able to generate much income since 1990. These observations are important for determining the measure of human well-being as reported in UNDPR for PNG in 2020 (PNG UNDP HDR,2020).

The raw data extracted from Table 1 are used to normalize the HDI data to obtain the current HDI for PNG at 0.555. This is computed in Table 2 using the geometric mean of health, education and income indices by substituting 2019 HDI data provided in Table 1 into the formula $HDI = \sqrt[3]{HI.EI.II}$.

Table 2: HDI data normalization on PNG HDI Data (2019)**.** Source: UNDP(2020b)

Sub-indicator	**Formula**	**PNG Data(2019)**
Health Index (HI)	$H = \frac{LEI - 20}{85 - 20}$	$H = \frac{64.5 - 20}{85 - 20} = \frac{44.5}{65} = 0.68462$
Education Index (EI)	$E = \frac{{MYS}/{15} + {EYS}/{18}}{2}$	$E = \frac{{4.7}/{15} + {10.2}/{18}}{2} = 0.44$
Income Index (II)	$I = \frac{\ln(GNIpc) - \ln(100)}{\ln(75000) - \ln(100)}$	$I = \frac{\ln(4301) - \ln(100)}{\ln(75000) - \ln(100)} = 0.56819$
Human Development Index	$HDI = \sqrt[3]{HI.EI.II}$	$HDI = \sqrt[3]{(0.68462).(0.44).(0.56819)} =$ **0.55522**

Notice that the same formulation from Table 2 can be used for computing the HDI for future years when data becomes available. For the years 2020 through to 2050, the PNG Government is aspiring for economic growth at some desirable rates underpinned by key development projects including better service delivery, improved education by promotion of human capital development, improved health services and sound political leadership and structures (PNG Vision 2050). We can observe the reality of this statement through an optimal solution of the model nesting the HDI of PNG with great emphasis on the educational attainment of the nation's entire population.

Based on the trends observed from the statistics in Table 1, the following predictions can be made for the remaining 3 decades leading up to 2050. First, life expectancy at birth has a growth rate of 3 years per decade. Expected years of schooling has grown by 2-3 years per decade. Mean years of schooling has a slow growth rate of 1 per decade. Finally, we have the gross national income per capita at a low growth rate. We formalise this in table 3 and make fair predictions for the future, which may affect the HDI to rise to the top 50 human development group.

Table 3: Predicted normalization HDI data for 2020-2050

Year	**Life expectancy at birth**	**Expected years of schooling**	**Mean years of schooling**	**GNI per capita($)**	**HDI value**
2020	65	10.5	4.9	4,500	0.566
2030	68	12.5	5.9	6,500	0.615
2040	71	14.5	6.9	8,500	0.689
2050	74	16.5	7.9	10,500	0.771

The model

We follow the DEA-like approach where component indices contribute positively to the HDI. Despotis (2005) suggest an output-oriented DEA model by assuming constant return-to-scale where all individual indicators are considered as outputs and a dummy input (equal to one) is assumed for all countries of study. Moreover, the weights are constrained to a sum less than or equal to 1. Unlike the HDI, the DEA-like approach to the assessment of human development is a relative measure. Hence PNG is compared with best-practice neighbouring countries when assessing its composite performance on the human development indicators weights (Despotis, 2005). We now formulate a simplified index maximizing LP model, which will be used to estimate an ideal value of the HDI of PNG and neighbouring countries. These calculations are then extended through a goal programming (optimization) model to derive a new measure of human development. We assume that the PNG economy has a level of education attainment high enough so that neither income nor illiteracy or poverty rates affect the life expectancy and income per capita as measured in the HDI. Hence, we maximize expenditures on health and income, whilst minimizing illiteracy and poverty rates. We begin by constructing a linear model based on this assumption.

We represent variables of the HDI as follow;

let C be the set of the 4 countries of the study, $j \in C$ stand for any country in C and j_0 stand for the evaluated country;

let w_{HI}, w_{EI}, w_{II} be the unknown weights of the three indices HI, EI and II respectively.

The linear model (1) below estimates the weights w_{HI}, w_{EI} and w_{II} that maximizes the weighted sum of three components of the HDI, for the evaluated country j_0, and it is solved for one country at a time. The weighted sum of the component indices is constrained to be less or equal to one for all countries. The infinitesimal ε ensures that none of the weights takes a zero value (Despotis, 2005).

$$max \;\; h_{j_o} = w_{HI}HI_{j_0} + w_{EI}EI_{j_0} + w_{II}II_{j_0} \quad (1)$$

s.t

$$w_{HI}HI_j + w_{EI}EI_j + w_{II}II_j \leq 1, j \in C,$$

$$w_{HI}, w_{EI}, w_{II}II_j \geq \varepsilon.$$

Model (1) is equivalent to an input-oriented, constant returns-to-scale DEA model with three outputs (HI, EI and II) and one dummy input of 1 for all countries (Mahlberg and Obersteiner,2001). Let h_j be the optimal value of the objective function when the model (1) is solved for country j. In this study, we have PNG and its neighbours Australia, Hongkong and Singapore as points of reference for a high human development group. The values h_j are bounded in the interval $[0,1]$. In notation, we let the objective function as a mapping $f: [0,1] \to \mathbb{R}$ defined by the function (1). A country that achieves a score of $h_j = 1$ is called an "efficient unit" in the DEA terminology and the opposite if the score is $h_j < 1$, the country j might be considered as "inefficient". Thus, if PNG has a low valued h_j in model (1), then it

shows a poor performance on human development and such calls for improved policies to cater for increment in the development indicators. The results from this analysis are given in Tables 4 and 5 below. Table 4 is based on 2019 data provided by UNDP report 2020 whilst table 5 is based on the predicted values from table 3.

Table 4: DEA-score for PNG against its neighbouring developed countries.

Country	HDI Rank	HDI	DEA Score
Hong Kong	(4)	0.949	1.000
Australia	(8)	0.944	1.000
Singapore	(11)	0.938	1.000
PNG	(155)	0.555	0.872

PNG's neighbouring developed countries suit well the "best practice" country and "efficient unit" as the DEA score $h_j = 1$ from table 4.

Table 5: DEA-score for PNG (2020-2050) based on predicted values.

Year	2020	2030	2040	2050
DEA Score	0.878	0.919	0.959	1

From table 5, PNG can be referred to as an "efficient unit" between 2020-2040 as the computed DEA score $h_j < 1$, but becomes an "efficient unit" by 2050 as $h_j = 1$.

Linear programming

We formulate the planner's problem from the objective function (1) by rewriting the variables into a matrix. The objective function (2) maximizes the HDI 30-years (2020-2050). The *lpSolve* package from R is used to solve the planner's problem (2). lpSolve contains routine lp (…) to solve linear optimization problems.

$$max\ \ h_{j(PNG)} = max_{w_{HI,}w_{EI},w_{II}}\, w_{HI}HI + \ w_{EI}EI + \ w_{II}II = \begin{bmatrix} HI \\ EI \\ II \end{bmatrix}^T \begin{bmatrix} w_{HI} \\ w_{EI} \\ w_{II} \end{bmatrix} \quad (2)$$

s.t

$$max_{w_{HI,}w_{EI},w_{II}}\, w_{HI}HI + \ w_{EI}EI + \ w_{II}II = \begin{bmatrix} HI \\ EI \\ II \end{bmatrix}^T \begin{bmatrix} w_{HI} \\ w_{EI} \\ w_{II} \end{bmatrix} \leq 1.$$

When function (2) is coded, the raw data for 2019 from Table 1 (Figure 2) and approximate values for 2050 from Table 3 (Figure 3) above are substituted as coefficients of the objective function with respect to the weights $w_{HI,}w_{EI}, w_{II}$ to determine the optimal values respectively. The optimal element obtained after solving the objective function (1) through linear programming is now 0.872 as shown in Figure 2.

```
# Import lpSolve package
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.1.1

# Set coefficients of the objective function. The coefficients are derived
from the life expectancy at birth(LEI), expected(EYS) and mean (MYS) of
schooling plus GNI per ca-pita data for the year 20....
f.obj <- c(64.5,0.88,4301)#values can be changed here.
# Set matrix corresponding to coefficients of constraints by rows #according
to predicted values for LEI, EYS & MYS with GNI per capita data #for the year
2050.
# Do not consider the non-negative constraint; it is automatically #assumed
f.con <- matrix(c(74,1.52,12000), nrow = 1, byrow = TRUE)
# Set unequality signs
f.dir <- c("<=")
# Set right hand side coefficients
f.rhs <- c(1)
#Optimal values of LEI, EDI,II
# Variables final values
lp("max", f.obj, f.con, f.dir, f.rhs)$solution

## [1] 0.01351351 0.00000000 0.00000000

#Objective at maximum
lp("max", f.obj, f.con, f.dir, f.rhs)

## Success: the objective function is 0.8716216
```

Figure 2: Objective function solved to a maximum value rounded to:0.872**.**

The general syntax of linear programming in R shown in Figure 3 means the following; *direction* controls whether to minimize or maximize, coefficients **c** are encoded a vector *objective.in*. Constraints A are given as a matrix *const.mat* with directions *const.dir*. Constraints **b** are inserted as a vector *const.rhs.*

```
lp(direction="min", objective.in, const.mat, const.dir,
   const.rhs)
```

Figure 3: General syntax of linear programming in R.

Results and discussion

The linear program coded in R solved the objective function derived from equations (1) - (2) by obtaining a maximized value of $h_{j(PNG)}$up to 0.872, that is: *##Success: the objective function is 0.872*. However, some observations can be made from the line of code in Figure 4. This is where the values for LEI, EYS and MYS plus per capita GNI can be changed for each year as data becomes available in future.

```
# Set coefficients of the objective function. The coefficients are derived
from the life expectancy at birth(LEI), expected(EYS) and mean (MYS) of
schooling plus GNI per ca-pita data for the year 20....
f.obj <- c(62,0.81,3661)#values can be changed here.
```

Figure 4: Coefficients of constraints.

When setting the matrix corresponding to the coefficients of constraints, the coefficient values can vary between 0 and 1 resulting in an arbitrary small number ε. This is possible because the objective function is defined on the mapping $f: [0,1] \rightarrow \mathbb{R}$. For instance, observe

the change in Figure 5 after inserting new values for LEI, EYS, MYS and per capita GNI from 2010 data (UNDPR.2020) with respect to w_{HI}, w_{EI}, w_{II} to maximize h_j.Such alternates in the coefficients represent individual values of the composite indicators(HI, EI, II) that make up the HDI.

```
# Import lpSolve package
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.1.1

# Set coefficients of the objective function. The coefficients are derived
from the life expectancy at birth(LEI), expected(EYS) and mean (MYS) of
schooling plus GNI per ca-pita data for the year 2010.
f.obj <- c(62,0.81,3661)
# Set matrix corresponding to coefficients of constraints by rows #according
to predicted values for LEI, EYS & MYS with GNI per capita data #for the year
2050.
# Do not consider the non-negative constraint; it is automatically #assumed
f.con <- matrix(c(74,1.52,12000), nrow = 1, byrow = TRUE)
# Set unequality signs
f.dir <- c("<=")
# Set right hand side coefficients
f.rhs <- c(1)
#Optimal values of LEI, EDI,II
# Variables final values
lp("max", f.obj, f.con, f.dir, f.rhs)$solution

## [1] 0.01351351 0.00000000 0.00000000

#Objective at maximum
lp("max", f.obj, f.con, f.dir, f.rhs)

## Success: the objective function is 0.8378378
```

Figure 5: Coefficients of constraints can be altered.

Conclusion

The index-maximized LP model was used to generate a new human measure called DEA-score apart from the HDI. PNG neighbouring countries namely Australia, Hong Kong and Singapore were used as best-practice countries to determine whether PNG can be termed as an "efficient" or "inefficient". The result obtained from the PNG DEA-score h_j was less than 1. Hence, PNG can be termed as an "inefficient" country in terms of human development. Moreover, linear programming in R statistical software using *lpSolve* library was used to solve the optimization problem where the result obtained is $h_{j(PNG)} = 0.872$, which falls between 0.7 and 0.9 indicating the expected PNG HDI value by the year 2050, thus achieving its' vision 2050 goal. Thus, PNG has the potential to be ranked as one of the top 50 high human development countries by 2050, which was proven through solving the optimization LP model.

Glossary

UNDP – United Nations Development Programme
HDI – Human Development Index
GNI – Gross National Income
PNG DSP – Papua New Guinea Development Strategic Plan

MTDP – Medium Term Development Plan
LP – Linear Programming
LEI – Life Expectancy Index
EI – Education Index
II – Income Index
BoD-Benefit of Doubt
DEA- Data Envelopment Analysis
HDI_{BOD} –Human Development Index Benefit of Doubt
MBoD-Multiplicative Benefit of Doubt
PNG- Papua New Guinea

Appendices

R-Source Code for linear programming

```
# Import lpSolve package
library(lpSolve)
# Set coefficients of the objective function. The coefficients are derived from the life expectancy at birth(LEI), expected(EYS) and mean (MYS) of schooling plus GNI per ca-pita data for the year 20....
f.obj <- c(64.5,0.88,4301)#values can be changed here.
# Set matrix corresponding to coefficients of constraints by rows #according to predicted values for LEI, EYS & MYS with GNI per capita data #for the year 2050.
# Do not consider the non-negative constraint; it is automatically #assumed
f.con <- matrix(c(74,1.52,12000), nrow = 1, byrow = TRUE)
# Set unequality signs
f.dir <- c("<=")
# Set right hand side coefficients
f.rhs <- c(1)
#Optimal values of LEI, EDI,II
# Variables final values
lp("max", f.obj, f.con, f.dir, f.rhs)$solution
#Objective at maximum
lp("max", f.obj, f.con, f.dir, f.rhs)
```

References

Gebo, R. & Anderson, P. K. (2019). Human Development Index: PNG progress and a mathematical explanation. *Electronic Journal of Informatics, 1,* 90–105.

The National (2021). *Dept launches development plans to meet Vision 2050*. Retrieved July 14 2021, from https://www.thenational.com.pg/dept-launches-development-plans-to-meet-vision-2050/

Despotis, D. K. (2005a). A reassessment of the human development index via data envelopment analysis. *Journal of the Operational Research Society, 56*(8), 969–980. https://doi.org/10.1057/palgrave.jors.2601927

Despotis, D. K. (2005b). Measuring human development via data envelopment analysis: The

case of Asia and the Pacific. *Omega, 33*(5), 385–390. https://doi.org/10.1016/j.omega.2004.07.002

Despotis, D. K., Stamati, L. V., & Smirlis, Y. (2010). Data envelopment analysis with nonlinear virtual inputs and outputs. *European Journal of Operational Research, 202*(2), 604–613. https://doi.org/10.1016/j.ejor.2009.06.036

Fox, W. P. (2020). *Nonlinear optimization: Models and applications*. Chapman and Hall/CRC: New York.

Investopedia. (2020). *Human Development Index (HDI)*. Retrieved July 14, 2021, from https://www.investopedia.com/terms/h/human-development-index-hdi.asp

Mahlberg, B. & Obersteiner, M. (2001). Re-measuring the HDI by data envelopment analysis. International Institute for Applied Systems Analysis (IIASA). *Interim Report IR-01–069*, Luxemburg, Austria. Doi:https://doi.org/10.2139/ssrn.1999372

Mariano, E. B., Ferraz, D., & de Oliveira Gobbo, S. C. (2021). The Human Development Index with multiple data envelopment analysis approaches: A comparative evaluation using social network analysis. *Social Indicators Research, 157(2), 1 - 58.*

Engineer, M., King, I. & Roy, N. (2008). The human development index as a criterion for optimal planning. *Indian Growth and Development Review, 1*(2), 172-192.

PNG's Millennium Development Goals Experience (2015). *Lessons learnt for the sustainable development goals*. Department of National Planning and Monitoring, Government of PNG: Port Moresby.

UNDP. (2020). *PNG Human Development Report 2020.* Retrieved 10 November 2021, from https://www.undp.org.

Yekta, A. P., Kordrostami, S., Amirteimoori, A., & Matin, R. K. (2018). Data envelopment analysis with common weights: the weight restriction approach. *Mathematical Sciences, 12*(3), 197-203.

Tabian Ambang. (2012). Progress and the challenges of implementing PNG Vision 2050: Reflections from Human Development Index ranking for 2011–2012. *Contemporary PNG Studies: DWU Research Journal 17,* 83-93.

Toffalis, C. (2013). An automatic-democratic approach to weight setting for the new human development index. *Journal of Population Economics, 26*(4), 1325–1345.

Acknowledgement

This paper is based on Michaelynne's final year research project. She was supervised by Raunu, who spent a lot of time assisting her and finding ways to solve the objective function of the optimization model. They wish to thank Cyril Sarsoruo for verifying the mathematical notations involved and Dr Martin Daniel (PhD) for reviewing this paper. However, responsibility for any errors of fact or opinion remains with the authors.

Authors

Michaelynne Yaguro is an undergraduate year 4 student of Mathematics and Computing Science at DWU. Email: myaguro21@gmail.com

Raunu Gebo Sarsoruo is a lecturer in the Department of Mathematics and Computing Science at DWU. Email: rgebo@dwu.ac.pg

Variation of the value of Pi on non-Euclidean surfaces

Peter K. Anderson

Abstract

Pi, the ratio of circumference to diameter of a circle, is an infinite non-repeating decimal number calculated, so far, to a trillion decimal places. This ratio was considered to be a universal constant until new geometries of curved surfaces were developed in the 19^{th} c. with the famous geometry of flat surfaces developed by Euclid (4^{th} c. BC) being but one example. In these non-Euclidean geometries, shortest paths are not straight lines, but great circles for spherical surfaces with positive curvature and hyperbolae for surfaces with negative curvature. This paper shows that on spheres, Pi becomes smaller as circle circumferences grow larger with the reverse occurring on hyperbolic surfaces. This topic is of general interest, given the worldwide celebration of Pi day on $3/14/xx$ each year when we try to interest the general population, including students in Madang schools, colleges and universities in mathematics.

Keywords: circumference, diameter, geometry, hyperbolic, non-Euclidean, Pi.

Introduction

Interest in Pi follows renewed focus on International Day of Mathematics Day (IDM) celebrated annually on March 14 as a worldwide celebration when all countries are "invited to participate through activities for both students and the general public in schools, museums, libraries and other spaces" (IDM, 2020, para. 1).

This paper will attempt to assemble already developed accessible algebraic proofs to demonstrate that the values of Pi decrease with increasing radius for circles on the surface of a sphere and increase on hyperbolic surfaces. We approach this both by considering the ratio of circle circumference to the radius on each surface type and also by considering the ratio of circle area to the radius on each surface (IDM, 2020).

Preliminary

We note that ordinary trigonometric functions $\sin\theta, \cos\theta$, and $\tan\theta$ form points on a unit circle. In hyperbolic geometry, we have corresponding functions $\sinh(x), \cosh(x)$ and $\tanh(x)$, which represent points on the right half of an equilateral hyperbola, hyperbolic sine and hyperbolic cosine. Whilst θ refers to angles subtended at the centre of a circle, x refers to positions on the x axis (Figure 1).

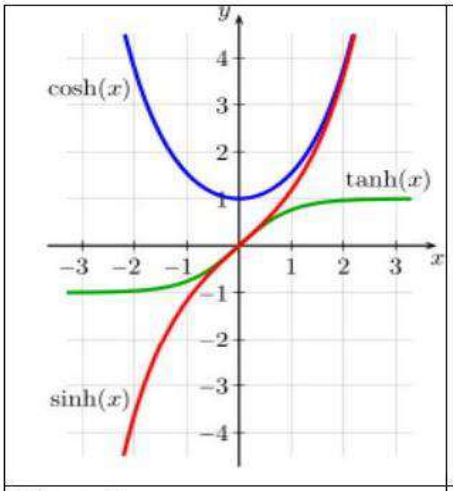


Figure 1
Source:
https://brilliant.org/wiki/hyperbolic-trigonometric-functions/

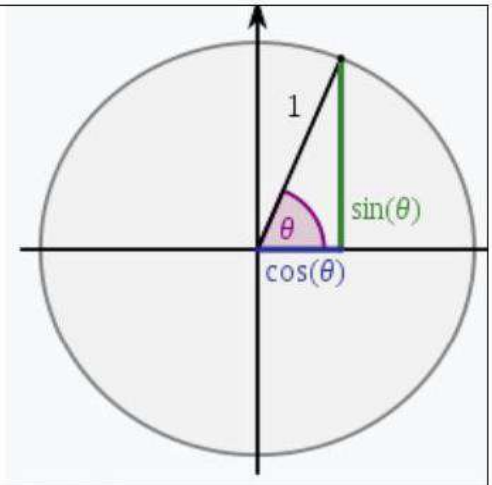


Figure 2

https://en.wikipedia.org/wiki/Trigonometric_functions

Figures 1 and 2: Circle perimeter on hyperbolic surfaces

The working of this section follows the method of UoG (n.dc). We consider a circle on a hyperbolic surface (Figure 3) where the shortest distance between any two points is a hyperbola. A triangle is shown as the first of an infinite number of equal area triangles to form an inscribed hyperbolic polygon to cover the circle.

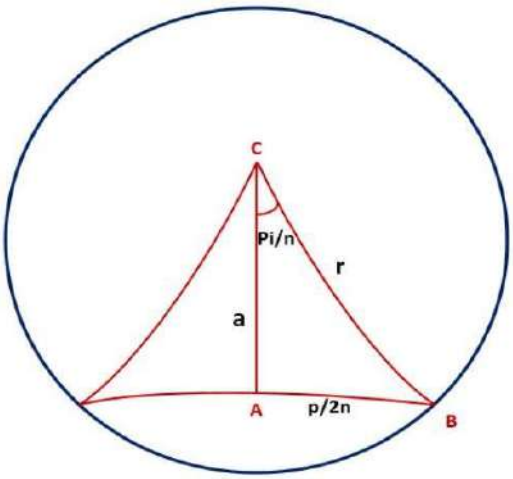


Figure 3 Showing a triangle on a hyperbolic surface where the shortest distance between any two points is a hyperbola.

Let p be the hyperbolic perimeter of an inscribed polygon consisting of n equal-area hyperbolic triangles, one of which is shown in Figure 3 . Let d be the area of the hyperbolic polygon (Weisstein, n.d).

We consider the polygon to be composed of 2n triangles CAB, each of area d/2n. We now use the well-known triangle sine rule for Δ CAB (Figure 3):

$$a/\sin A = b/\sin B = c/\sin C$$

which becomes for ΔCAB where $\sin A = 1$, since A is a right angle:

$$a = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Thus we can writes

$$\sin C = \frac{c}{a}$$

which can be written for Figure 3 as:

$$\sin\left(\frac{\pi}{n}\right) = \sinh\left(\frac{p}{2n}\right)/\sinh(r) \tag{1}$$

since x maps to $\sinh(x)$ on a hyperbolic surface where $\sinh(x)$ is the shortest distance between two points on the surface.

For n triangles, we can now write:

$$n\sin\left(\frac{\pi}{n}\right) = n\sinh\left(\frac{p}{2n}\right)/\sinh(r) \tag{2}$$

To proceed further, we need the Limit Lemma.

Limit lemma

The working of this section also follows the method of UoG (n.db). Here we prove the limit lemma for the sin and sinh functions which states that:
If the function f is differentiable at 0 and $k \neq 0$, then: $nf(k/n) \to kf(0)$ as $n \to \infty$.

Proof: We use the well-known definition for differentiation.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

which, at $x = 0$, becomes:

$$f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

We can now replace *h* with *x* a position on the *x*-axis, since sin and sinh functions are both zero at $x = 0$, to derive:

$$f'(0) = \lim_{x\to 0} \frac{f(x)}{x}.$$

We now introduce constant $k \neq 0$ such that $kx \to 0$ as $x \to 0$ Thus

$$f'(0) = \lim_{x\to 0} \frac{f(kx)}{kx}$$

and so

$$kf'(0) = \lim_{x\to 0} \frac{f(kx)}{x}.$$

Now let $k = 1/n$, with $n \to \infty$ as $x \to 0$ then

$$nf'\left(\frac{k}{n}\right) \to kf'(0) \text{ as } n \to \infty.$$

For $f(x) = \sin x$ or $f(x) = \sinh x$ we have:

$$f'(0) = \cos 0 = 1$$
$$f'(0) = \cosh 0 = 1$$

so we can conclude:

$$f\left(\frac{k}{n}\right) \to k \text{ as } n \to \infty. \quad (3)$$

Incorporating the limit lemma

We can now proceed using (3). As $n \to \infty, p \to C(r)$, the hyperbolic perimeter, and so equation (2) becomes:

$$n\pi/n\sinh(r) = n\frac{p}{2n}$$
$$\pi/\sinh(r) = \frac{p}{2}$$
$$\text{or } p = 2\pi\sinh(r), \text{ and}$$
$$\pi\sinh(r) = \frac{C(r)}{2} \text{ with}$$
$$C(r) = 2\pi\sinh(r), \text{ the hyperbolic perimeter.}$$

Variation of Pi on a hyperbolic surface

We now consider the value of Pi for a circle on a hyperbolic surface and let π ' be the value of π (ratio of circumference to diameter) on a hyperbolic surface. We have:

$$\pi\sinh(r) = \pi' r$$

and so

$$\pi' = \pi\sinh(r)/r. \quad (4)$$

Variation of Pi on a spherical surface

The working of this section follows the method Maximenko (2015). We now consider a spherical surface and the Δ CAB (Figure 3).

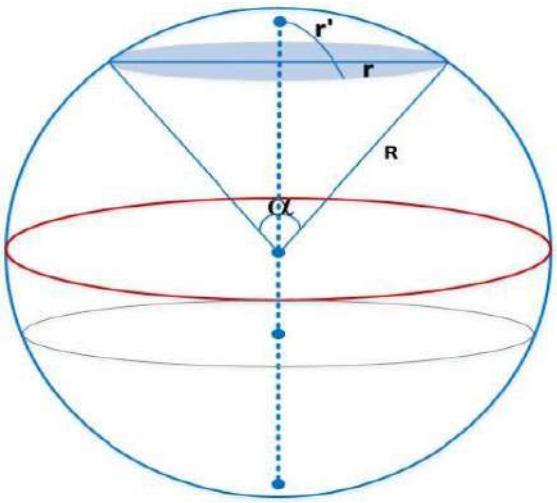


Figure 3 3D sphere with circles of surface radius r ' and plane radius r on its surface.

$$\text{Sin } \alpha/2 = r/R \quad (5)$$

from trigonometry and by definition of radian angle measure, we have:

$$\text{Angle in radians} = \text{subtending arc / radius}$$
$$= \mathrm{r}'/\mathrm{R} \text{ for } \Delta\mathrm{CAB}$$
$$= \frac{\alpha}{2} . \tag{6}$$

From (5) and (6) we can write:

$$\frac{r'}{R} = \frac{\alpha}{2} / \sin\left(\frac{\alpha}{2}\right)$$

and, since $r < r'$:

$$r/r' = \sin\left(\frac{\alpha}{2}\right) / \left(\frac{\alpha}{2}\right) < 1.$$

For a spherical surface, circle circumference is $2\pi r = 2\pi' r'$.

Where, again, π ' and r' refer to the surface values. Thus, we have an expression for the variation of π with surface radius:

$$\begin{aligned} \pi' = \quad & \pi r/r' < \pi \\ & = \pi \sin\left(\frac{\alpha}{2}\right) / \left(\frac{\alpha}{2}\right) \\ & = \pi \sin\left(\frac{r}{R}\right) / \left(\frac{r}{R}\right) \\ & = \pi \sin(r)/(r) \end{aligned} \tag{7}$$

for a sphere with unit radius, $R = 1$.

Comparing Pi variations

Using equations (4) and (7) we can compare the variation of Pi on each of the two types of surfaces discussed in this paper as the surface radius of circles is increased. Whilst the value of Pi on a plane surface is constant and does not vary with circle size, Pi increases rapidly for a hyperbolic surface and decreases for a spherical surface.

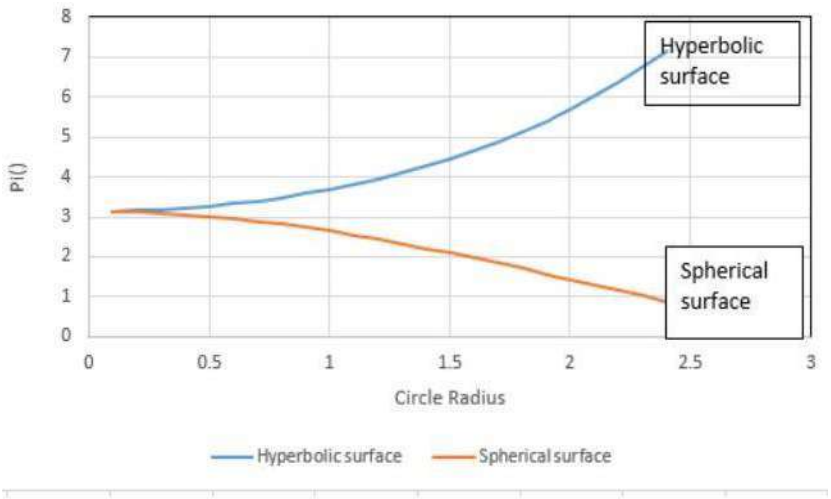


Figure 4 Using equations (4) and (7) we can compare the variation of Pi on each of the two types of surfaces discussed in this paper as surface radius is increased.

Surface area of a circle on a hyperbolic surface

Again, we consider a triangle on a hyperbolic surface (Figure 3) multiple occurrences of which will cover the whole circle area (UoG, n.da). From Heron's formula for triangle areas adapted for hyperbolic triangles:

$$\sin\left(\frac{d}{n}\right) = \sinh(a)\sinh\left(\frac{p}{2n}\right)/(\cosh((r)+1)$$

where d is triangle area, and for 2n triangles we have:

$$2n\sin\left(\frac{d}{n}\right) = \sinh(a)2n\sinh\left(\frac{p}{2n}\right)/(\cosh((r)+1).$$

Let D(r) and C(r) be the hyperbolic circle area and circumference respectively. As $n \to \infty$, $D(r) = d$, the hyperbolic circle area, $a \to r, \&$ and $C(r) = p$ the hyperbolic circumference. We can now write:

$$\begin{aligned} D(r) &= (\sinh(r)C(r))/(\cosh(r)+1) \\ &= (\sinh(r)2\pi\sinh(r))/(\cosh(r)+1) \\ &= 2\pi\sinh^2\left(\frac{r}{2}\right)\cosh^2\left(\frac{r}{2}\right)/\left(2\cosh^2\left(\frac{r}{2}\right)\right) \\ &= 4\pi\sinh^2\left(\frac{r}{2}\right) \end{aligned}$$

Thus, area of a circle on a hyperbolic surface is given by:

$$D(r) = 4\pi\sinh^2\left(\frac{r}{2}\right). \quad (8)$$

Surface area of a circle on a spherical surface

The working of this section follows the method of StackExchange (2016). Referring to Figure 5 we have:

$$\alpha = \frac{r}{R} \text{ and } \sin\alpha = \frac{x}{R}$$
$$\sin\left(\frac{r}{R}\right) = \frac{x}{R}$$
$$x = R\sin\left(\frac{r}{R}\right).$$

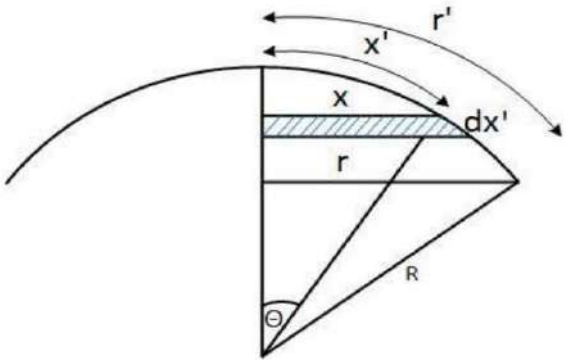


Figure 5 The area of a cap on a sphere is calculated by considering a thin disk of radius x and thickness dx' and then integrating the disk surface strip from 0 to r '.

Area of circular surface strip $= 2\pi x dx'$
Area of cap,

$$= 2\pi R^2 \int_0^{r'} \sin y dy \text{ where y} = x'/R \& R dy = dx' \qquad \begin{aligned} \text{A} &= 2\pi R \int_0^{r'} \sin \frac{x'}{R} dx' \\ &= 2\pi R^2 [-\cos y]_0^{r'/R} \\ &= 2\pi R^2 \left(1 - \cos\left(\frac{r'}{R}\right)\right) \\ &= 2\pi(1 - \cos r') \end{aligned}$$

for $R = 1$ on a unit sphere.
Thus, area of a circle on a spherical surface is given by:

$$D(r) = 2\pi(1 - \cos r'). \tag{9}$$

Comparing circle area variations

Using equations (8) and (9) we can compare the variation of circle area on each of the two types of surfaces discussed in this paper as the surface radius of circles is increased. Disks areas grow faster with length of surface radius on plane surfaces than on spherical surfaces and faster again on hyperbolic surfaces.

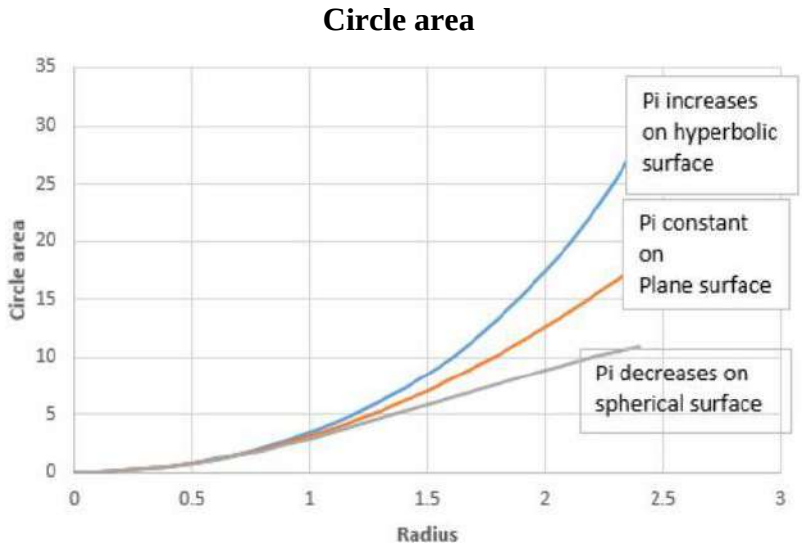


Figure 5 Disks areas grow faster with length of surface radius on plane surfaces than on spherical surfaces and faster again on hyperbolic surfaces.

Conclusion

This paper has assembled algebraic equations to demonstrate that the values of Pi decrease with increasing radius for surface circles on a sphere and increase on hyperbolic surfaces. This has been approached both by considering the ratio of circle circumference to the radius on each type of surface and also by considering the ratio of circle area to the radius on each surface.

References

International Day of Mathematics. (2020). *About the International Day of Mathematics* Retrieved 20 November 2020, from https://www.idm314.org/about.html.

Maximenko, Y. (2015). *Pi is not as constant as you think!* Retrieved 15 November 2020, from https://physics.illinois.edu/news/34508.

StackExchange (2016). *Mathematics: Area of a circle on sphere* Retrieved 10 November 2020, from https://math.stackexchange.com/questions/1832110/area-of-a-circle-on-sphere/1832144#1832144.

UoG. (n.da). *Hyperbolic area - heron's formula* Retrieved 10 November 2020, from https://www.maths.gla.ac.uk/wws/cabripages/hyperbolic/harea2.html.

UoG. (n.db). *Some useful limits* Retrieved 10 November 2020, from https://www.maths.gla.ac.uk/wws/cabripages/hyperbolic/limits.html

UoG. (n.dc). *The circumference and area of hyperbolic circles* Retrieved 10 November 2020, from https://www.maths.gla.ac.uk/wws/cabripages/hyperbolic/circleformulae.html.

Weisstein, E. W. (n.d). Hyperbolic Geometry Retrieved 14 November 2020, from https://mathworld.wolfram.com/HyperbolicGeometry.html.

Acknowledgement

I would like to thank Dr Martin Daniel and Cyril Sarsoruo for reviewing this paper. However, responsibility for any errors of fact or opinion remains with the author.

Author

Peter K. Anderson (late) was a Professor in the Faculty of Business & Informatics at the Divine Word University. He was the founder and Head of the Department of Information Systems. He also served as the Head of the Department of Mathematics & Computing Science until his untimely death in Feb 2021.

Computing cube root of a real number

Ram Bilas Misra
Ranjana Bajpai

Abstract

Computing square root of a real number of any digits (whether involving integral part only or both integral as well as decimal parts) is taught in lower classes in the Indian sub-continent but nowhere else outside. It has been a matter of great curiosity why computation of roots of higher order of a real number has never been taught? We formulated a method to compute the real cube root of a real number irrespective of its nature whether containing only an integral part or only decimal part or both.

Keywords: cube root, mathematical computation, real numbers.

§1. Cube root of four digits (whole) numbers which are perfect cubes of some integers

Let x_1, x_2, x_3 be the digits at the respective places of unity, ten, hundred assuming values from 0 to 9 and x_4 the digit at thousand's place (running from 1 to 9). Thus, writing the number as $x_4\ x_3\ x_2\ x_1$ its numerical value is

$$10^3.\ x_4 + 10^2.\ x_3 + 10.\ x_2 + x_1.$$

Rule 1.1. We group the digits of number starting from the unit's digit in triples, i.e. x_1, x_2, x_3 in the first group and the next group consists of the single digit x_4. Now, we look for the largest (whole) number a (say), whose *cube* does not exceed the digit x_4. As x_4 assumes values from 1 to 9, a can have only two choices: 1 and 2.

Next, we carry out the division of the given number ($x_4\ x_3\ x_2\ x_1$) by a^2 as illustrated below:

$$\begin{array}{r|l} & a \\ \hline a^2 & x_4x_3x_2x_1 \\ & -a^3 \\ \hline & x^4 - a^3 \equiv x_5 \end{array} \qquad (1.1)$$

(say) to its right carry over the triple $x_3\ x_2\ x_1$ to get $x_5\ x_3\ x_2\ x_1$. Now, we choose the largest digit b (to be placed to the right of a in quotient) so that the cube of the number $a\ b$ (with numerical value $10\ a + b$):

$$(10\ a + b)^3 \equiv 1000\ a^3 + (3 \times 100\ a^2 \times b + 3 \times 10\ a \times b^2 + b^3) \qquad (1.2)$$

does not exceed the given number. In other words, the number within the parentheses on RHS of Eq. (1.2), i.e. $\{3.(100\ a^2 + 10\ a.\ b) + b^2\}.b$ does not exceed the remainder $x_5x_3x_2x_1$ obtained in Eq. (1.1). Thus, the second divisor

$$c \equiv 3.(100\ a^2 + 10\ a.\ b) + b^2 \qquad (1.3)$$

divides $x_5x_3x_2x_1$ by b times:

$$\begin{array}{r|l} & ab \\ \hline 3\cdot(100\ a^2 + 10\ a\cdot b) + b^2 & x_5x_3x_2x_1 \\ & -\{3(100\ a^2 + 10\ a\cdot b) + b^2\}\cdot b. \end{array} \qquad (1.4)$$

If the remainder in the above division process in Eq. (1.4) is zero, the desired cube root is the number *a b* (with numerical value 10 *a* + *b*). //

Example 1.1. To compute the cube root of the number 1,728.

Soln. Proceeding as above, we carry the division. After the first process of division, we sought the digit *b* (to be placed to the right of quotient 1). Taking it as 1, the divisor *c* will be 3. $(100 \times 1^2 + 10 \times 1 \times 1) + 1^2 = 331$ whose product with *b* (i.e. 1) is much lower than 728. Repeating the exercise with $b = 2$, we get $c = 3$. $(100 \times 1^2 + 10 \times 1 \times 2) + 2^2 = 364$ whose product with $b = 2$ makes exactly 728.

$$
\begin{array}{r|l}
 & 12 \\
\hline
1^2 & 1{,}728 \\
 & -1 \\
\hline
364 & 728 \\
 & -728 \\
\hline
 & 0.
\end{array}
$$

Thus, the cube root of the given number is 12. //

Example 1.2. To compute the cube root of the number 15,625.

Soln. Proceeding as above, we carry the division. Beginning with $b = 4$, we get $c \equiv 3$. $(100 \times 2^2 + 10 \times 2 \times 4) + 4^2 = 1{,}456$ whose product with $b = 4$ is lower than 7,625. Repeating the exercise with $b = 5$, we get $c = 3$. $(100 \times 2^2 + 10 \times 2 \times 5) + 5^2 = 1{,}525$ whose product with $b = 5$ makes exactly 7,625.

$$
\begin{array}{r|l}
 & 25 \\
\hline
2^2 & 15{,}625 \\
 & -8 \\
\hline
1525 & 7625 \\
 & -7625 \\
\hline
 & 0.
\end{array}
$$

Thus, the cube root of the given number is 25. //

Example 1.3. To compute the cube root of the number 42,875.

Soln. Proceeding as above, we carry the division: 3 is the largest digit whose cube does not exceed 42. Thus, 3^2 divides 42 by 3 times leaving the remainder 15. Borrowing the triple 875 to the right of 15 we look for the largest digit *b* to be placed to the right of $a = 3$ in the quotient so that the next divisor *c*, given by Eqn. (1.3), divides the number 15,875 by *b* times. The choice of $b = 5$ giving $c = 3$. $(100 \times 3^2 + 10 \times 3 \times 5) + 5^2 = 3{,}175$ divides the number exactly by 5 times.

$$
\begin{array}{r|l}
 & 35 \\
\hline
3^2 & 42875 \\
 & -27 \\
\hline
3175 & 15875 \\
 & -15875 \\
\hline
 & 0.
\end{array}
$$

Thus, the cube root of the given number is 35. //

Example 1.4. To compute the cube root of the number 91,125.

Soln. Proceeding as above, we carry the division. Here, $a = 4$ is the largest digit whose cube does not exceed 91. Thus, 4^2 dividing 91 by 4 times leaves the remainder 27. Borrowing the triple 125 the next dividend is 27,125. Again, we look for the largest digit b to be placed to the right of 4 in the quotient so that the next divisor c, given by Eqn. (1.3), divides the number 27,125 by b times. The choice of $b = 5$ makes $c = 3. (100 \times 4^2 + 10 \times 4 \times 5) + 5^2 = 5{,}425$ that divides the number exactly by 5 times.

$$
\begin{array}{r|l}
 & \quad 45 \\
\hline
4^2 & \quad 91125 \\
 & -64 \\
\hline
5425 & \quad 27125 \\
 & -27125 \\
\hline
 & \quad\quad 0.
\end{array}
$$

Thus, the cube root of the given number is 45. //

Example 1.5. To compute the cube root of the number 166,375.

Soln. Proceeding as above, we carry the division. Here, $a = 5$ is the largest digit whose cube does not exceed 166. Thus, 5^2 dividing 166 by 5 times leaves the remainder 41. Thereafter, borrowing the triple 375 the next dividend is 41,375. Exploring with $b = 4$ we would get $c = 3. (100{\times}5^2 + 10{\times}\ 5{\times}4) + 4^2 = 8{,}116$ whose product with $b = 4$ is much lower than 41,375. But, $b = 5$ gives $c = 3. (100 \times 5^2 + 10 \times 5 \times 5) + 5^2 = 8{,}275$ whose product with $b = 5$ makes exactly 41,375.

$$
\begin{array}{r|l}
 & \quad 55 \\
\hline
5^2 & \quad 166{,}375 \\
 & -125 \\
\hline
8275 & \quad 41375 \\
 & -41375 \\
\hline
 & \quad\quad 0.
\end{array}
$$

Thus, the cube root of the given number is 55. //

§ 2. Cube root of numbers having only decimal part but perfect cube

Let x_1, x_2, x_3, x_4 be the successive digits after the decimal towards the right forming the number $0{\cdot}x_1\ x_2\ x_3\ x_4$. Each one of the digits x_1, x_2, x_3, x_4 may assume values from 0 to 9. In this case, we group the successive digits x_1, x_2, x_3 in the first triple towards the right (unlike to Rule 1.1) and think of the largest (single digit) number, say a, with cube not exceeding the value of the number

$$10^2.\, x_1 + 10.\, x_2 + x_3 \equiv y_1 \quad \text{(say)}, \qquad\qquad (2.1)$$

formed by the digits x_1, x_2, x_3 and carry out the *first* process of division of the number y_1 by a^2 leaving quotient a and remainder $y_1 - a^3 = y_2$ (say). The second triple is formed by digit x_4 and additional two (borrowed) digits each of value 0 (zero), which is always permissible. This second triple with digits x_4, 0, 0 is placed to the right of remainder y_2 and the process of subsequent division(s), as explained above, after Eqn. (1.1), is followed. The noticeable point

here is that the role of decimal is almost ignored. However, it is placed before a in the quotient.

The process is elaborated through the following examples.

Example 2.1. To compute the cube root of the number 0·729.

Soln. The largest single digit with a cube not exceeding the number 729 (ignoring the decimal) is 9. Thus, taking $a = 9$, the division is carried by $a^2 = 81$:

$$\begin{array}{r|l} & \quad 0 \cdot 9 \\ \hline (0.9)^2 = 0 \cdot 81 & \quad 0 \cdot 729 \\ & -0 \cdot 729 \\ \hline & \quad\quad 0 \end{array}$$

determining the desired cube root 0·9. //

Example 2.2. To compute the cube root of the number 0·001331.

Soln. The first triple to the right of the decimal point is 001; so, here, $a = 1$. Division by $a^2 = 1$ leaves the quotient 1 and remainder 0. The next triple 331 is placed to the right of remainder 0 (forming the number 331). Now, we think of the single (largest) digit b, to be placed to the right of existing digit $a = 1$ in the quotient and the next divisor [cf. § 1]

$$c \equiv 3.\,(100\,a^2 + 10\,a.b) + b^2 \;=\; 3.\,(100 \times 1^2 + 10 \times 1 \times b) + b^2$$

divides 331 by b times. Such a choice of b is only 1 and the corresponding value of $c = 331$ divides the number 331 by once. Hence, the digits $a = 1$ and $b = 1$ determine the cube root 0·11.

$$\begin{array}{r|l} & \quad\quad 0 \cdot 11 \\ \hline (0.1)^2 = 0 \cdot 01 & \quad 0 \cdot 001\,331 \\ & -0 \cdot 001 \\ \hline 331 & \quad\quad\quad 331 \\ & \quad\quad -331 \\ \hline & \quad\quad\quad\; 0. \end{array}$$

Example 2.3. To compute the cube root of the number 0·015625.

Soln. The first triple to the right of the decimal point is 015; so, here, $a = 2$. Division by $a^2 = 4$ leaves the quotient 2 and remainder 7. The next triple 625 is placed to the right of remainder 7 (forming the number 7,625). Now, we think of the single (largest) digit b, to be placed to the right of existing digit $a = 2$ in the quotient, so that the next divisor c in Eqn. (1.3) may divide the number 7,625 by b times. Such a choice of b is 5, and corresponding c = 1,525 exactly divides 7,625 by 5 times. Hence, the desired cube root is 0·25.

$$
\begin{array}{r|l}
 & \quad 0\cdot 25 \\
\hline
(0.2)^2 = 0\cdot 04 & \quad 0\cdot 015625 \\
 & -0\cdot 008 \\
\hline
1525 & \quad\quad 7625 \\
 & \quad\quad -7625 \\
\hline
 & \quad\quad\quad 0.
\end{array}
$$

§ 3. Cube root of any whole number

The process is the same as explained in §1 and we continue up to Eqn. (1.4). Naturally, the remainder in Eqn. (1.4) is not *zero* here. Hence, we place a decimal point to the right of (given whole number) and borrow a triple formed by digits each of value 0 (zero) to be placed after the remainder. Simultaneously, a decimal point is also placed immediately in the quotient. Thereafter, the process as explained in §2 is followed. We demonstrate the process using some examples.

Example 3.1. To compute the cube root of the number 11.

Soln. Since there are only 2 digits in the integral part of the number we do not try to form a triple and just think of a single digit largest number with a cube not exceeding 11. Such a digit is 2. Thus, taking $a = 2$, the first process of division is carried by $a^2 = 4$ leaving the quotient 2 and the remainder 3. Thereafter, decimal points are simultaneously placed in the quotient as well as to the right of (given number) 11. Now, we borrow triple (000) with each zero digit and place it to the right of remainder 3 to form the dividend 3,000. We think of the largest single digit b to be placed in the quotient (after the decimal point) so that the corresponding value of next divisor c, given by Eqn. (1.3), may divide the number 3,000 by b times. Such a choice of b is 2 so that $c = 1{,}324$ divides 3,000 by 2 times leaving the remainder 352. Again borrowing the next triple 000 and placing it to the right of remainder 352 making the dividend 352,000. We think of the next single largest digit b to be placed to the right of $2\cdot 2$ in the quotient, so that the corresponding c may divide 352,000 by b times. Noting the current value of a as 22 (ignoring the decimal point in the quotient), $b = 2$ again meets the requirements and

$$c \equiv 3.(100 \times 22^2 + 10 \times 22 \times 2) + 2^2 = 146{,}524$$

divides 352,000 by 2 times, leaving the remainder 58,952. Again, borrowing the next triple 000 after the above remainder makes the dividend 58,952,000. Now, with $a = 222$, we look for the largest b so that the corresponding c can divide this new dividend. The choice $b = 3$ meets the requirements and

$$c \equiv 3.(100 \times 222^2 + 10 \times 222 \times 3) + 3^2 = 14{,}805{,}189$$

divides 58,952,000 by 3 times, leaving the remainder 14,536,433.

$$
\begin{array}{r|l}
 & \quad 2\cdot 223 \\ \hline
2^2 & \quad 11.000\ 000\ 000 \\
 & -\ 8 \\ \hline
1324 & \quad 3000 \\
 & -2648 \\ \hline
146524 & \quad 352000 \\
 & -293048 \\ \hline
14805189 & \quad 58952000 \\
 & -44415567 \\ \hline
 & \quad 14536433.
\end{array}
$$

Thus, computing up to 3 decimal places, the desired cube root is 2·223. //

§ 4. Cube root of a mixed number having both integral and decimal parts

For the cube root of an integral part, the process is the same as explained in §1 and we continue up to Eqn. (1.4). Again, the remainder in Eqn. (1.4) may not be *zero* in general. Hence, we place a decimal point to the right of the available quotient and borrow the available triple formed by the first three digits after the decimal point in the dividend to the right of the remainder. Thereafter, the process as explained in § 2 is followed.

We demonstrate the process using some examples.

Example 4.1. To compute the cube root of the number 4·096.

Soln. Since the integral part of the number has only one digit we ignore the concept of a triple and just think of a single digit largest number with a cube not exceeding the integral part. Such a digit is 1. Thus, taking $a = 1$, the first process of division is carried by $a^2 = 1$ leaving the quotient 1 and the remainder 3. Thereafter, a decimal point is placed in the quotient and the triple (096) available in the decimal part of the number is carried to the right of remainder 3 to form the dividend 3,096. We think of the largest single digit b to be placed in the quotient (after the decimal point) so that the corresponding value of next divisor c, given by Eqn. (1.3), may divide the number 3,096 by b times. Such a choice of b is 6 so that $c = 3.(100\times1^2 + 10\times1\times6) + 6^2 = 516$ divides 3,096 by 6 times exactly leaving zero remainder.

$$
\begin{array}{r|l}
 & \quad 1\cdot 6 \\ \hline
1^2 = 1 & \quad 4\cdot 096 \\
 & -\ 1 \\ \hline
516 & \quad 3096 \\
 & -3096 \\ \hline
 & \quad 0.
\end{array}
$$

Hence, the desired cube root is 1·6. //

Example 4.2. To compute the cube root of the number 1234·5678.

Soln. Following the method explained in §1, the first triple (counted from the decimal point towards left) is 234 and thereafter, there is only a single digit 1. As such, there exists only a number 1 whose square divides this single digit dividend 1 by 1. So, the first available digit in the quotient is also 1 and the remainder is zero. Thereafter, the next triple (234) is

borrowed to the right of remainder 0 to form the next dividend 0234, i.e. 234 only. Looking for the largest single digit b to be placed in the quotient so that the corresponding c given by Eqn. (1.3) divides 234 by b times. Such choice of b is only 0 for which c = 300 divides 234 by 0 times leaving the remainder again 234 and the quotient becomes 10. Thereafter, a decimal point is placed in the quotient and the triple (567) available in the decimal part of the number is borrowed to the right of remainder 234 to form the new dividend 234,567. The search for the next largest single digit b to be placed in the quotient (after the decimal point) is made so that the corresponding value of divisor c may divide 234,567 by b times. Such a choice of b is 7 making the quotient 10·7 and c = 32,149 that divides 234,567 by 7 times leaving the remainder 9,524. Again borrowing the next triple 800 and carrying it to the right of the remainder 9,524 making the dividend 9,524,800. Again, the next search for the single largest digit b to be placed to the right of the quotient 10·7, so that the corresponding c may divide 9,524,800 by b times. Noting the current value of a as 107 (ignoring the decimal point in the quotient), b = 2 meets the requirements and $c \equiv 3.(100 \times 107^2 + 10 \times 107 \times 2) + 2^2$ = 3,441,124 divides 9,524,800 by 2 times, leaving the remainder 2,642,552. The quotient gets supplemented as 10·72. Borrowing the next triple 000 to the right of the above remainder makes the dividend 2,642,552,000. Now, with a = 1,072, we look for the largest b so that the corresponding c can divide this new dividend. The choice b = 7 meets the requirements supplementing the quotient as 10·727 and $c \equiv 3.(100 \times 1072^2 + 10 \times 1072 \times 7) + 7^2$ = 344,980,369 divides 2,642,552,000 by 7 times, leaving the remainder 227,689,417.

$$
\begin{array}{rl}
 & 10\cdot 727 \\
\hline
1^2 & 1234\cdot 567\,800\,000 \\
 & -\,1 \\
\hline
300 & \quad 234 \\
 & \quad -0 \\
\hline
32149 & \quad 234567 \\
 & \quad -225043 \\
\hline
3441124 & \quad\quad 9524800 \\
 & \quad\quad -\ 6882248 \\
\hline
344980369 & \quad\quad 2642552000 \\
 & \quad\quad -\,2414862583 \\
\hline
 & \quad\quad\quad 227689417\,. \\
\hline
\end{array}
$$

Thus, computing up to 3 decimal places, the desired cube root is 10·727. //

Acknowledgements

The first author expresses his gratitude to one of his senior colleagues (late) Professor Dr. H. Gupta, Founder Professor & Head, Dept. of Mathematics, Punjab University, Chandigarh (India) – a Number Theorist of international repute for inspiration through his dedication and attachment to research despite his physical restraints of Parkinson's disease. Also, thanks are due to the Lebanese French University, Erbil, KRG (Iraq) for their generosity in providing the necessary facilities to accomplish the project. The main results of the paper were worked out during the first author's stay at Divine Word University, Madang (PNG) in 2015-16. Thus, the paper is dedicated to the memory of the late Prof. Peter K. Anderson.

References

Gupta, A. *Long division method of cube root*. Retrieved 31 August 2021, from https://brilliant.org/discussions/thread/long-division-method-of-cube-root/

Authors

Prof. Dr. Ram Bilas Misra
Ex Vice-Chancellor, *Avadh University*, *Ayodhya* (*India*);
Research and Strategic Studies Centre, *Lebanese French University*, *Erbil*, *KRG* (*Iraq*);
380 A, Gomti Nagar Extension, Sector 1, Lucknow – 226010, U.P. (India);
rambilas.misra@gmail.com, misrarb1@rediffmail.com

Mrs. Ranjana Bajpai
C 608, Hindon Society, Plot 25, Vasundhara Enclave, New Delhi – 110026, (India), Email: bajpairanjana@gmail.com

Primality and factorization - a computer challenge

S. P. Khare
Ram Bilas Misra

Abstract

The second author had a long association with Satgur Prasad Khare, as his student both in graduate and postgraduate classes at the University of Allahabad, Prayagraj (India) for over five decades. Khare worked in the Theory of Numbers under the supervision of a celebrated Indian Number Theorist (Prof. H. Gupta). Even after his retirement, he continued his researches in Number Theory and, recently, he developed an interest in cryptography. Due to the lack of senior workers in his field, he trusted his own teacher and often used to send his researches to the author for review. He met a tragic and untimely end of his life on 30 April 2021 leaving a lot of his unpublished work that needs to be highlighted. He was completing a research monograph on Number Theory before his sudden cardiac arrest. The present article deals with the *primality and factorization* of RSA numbers including the latest test called the *AKS Primality Test*. Results of number theory are comprehensible by even the non-specialists but their proofs are most challenging.

Keywords: mathematical computation, primality and factorization, RSA numbers.

§ 1. Preliminaries

Let N, Z and P denote the sets of natural numbers, integers and prime numbers:

$$\mathrm{N} = \{ 1, 2, 3, \ldots\}, \qquad \mathrm{Z} = \{ 0, \pm 1, \pm 2, \pm 3, \ldots \},$$

$$\mathrm{P} = \{2, 3, 5, 7, 11, \ldots \}.$$

Unless stated otherwise, small (Roman) letters shall denote positive integers. If $n = a\,b$, $a, b \in \mathrm{N}$, $a \neq 1$, $b \neq 1$, then a and b are called *proper divisors* of n. So every natural number has either one, two or more divisors. Numbers having only two divisors are known as *prime numbers* (or briefly primes), while others as *composite numbers*. As per the *Fundamental theorem of arithmetic*, every natural number is expressible as a product of (distinct powers of) primes uniquely:

$$n = (p_1)^{a_1}.(p_2)^{a_2}.(p_3)^{a_3}\ldots(p_k)^{a_k},$$

also called the canonical form of n.

1.1. Some concerning results/conjectures about primes

(i) There are two forms of primes viz. $4k + 1$ and $4k - 1$;

(ii) Number of primes is infinite;

(iii) There are infinite pairs of the form $(p, p + 2)$;

(iv) There are infinitely many primes of the form $(p, p + 2, p + 4)$ and $(p, p + 4, p + 6)$;

(v) Does there exist any (simple) general formula for n^{th} prime?

(vi) Is there any formula satisfying $p_{n+1} = (p_n)^2 + 2$?

(vii) Is there a rule determining another prime greater than the given prime p?

(viii) How many primes may exist, which are not exceeding a given number x?

(ix) If $n \geq 6$, then n is the sum of two odd primes which still requires a proof – called *Goldbach conjecture*.

(x) There is always a prime between n^2 and $(n + 1)^2$ but it requires proof.

Note 1.1. Though number theory is classical, problems are easy to understand but really challenging to prove.

Given two numbers a and b, Euclid found two more numbers: q and r such that

$$a = q\,b + r; \qquad 0 \leq r < b\,.$$

This is known as *Euclidean Algorithm* – The first algorithm is known in Number theory (Computer Science). In particular, it is an algorithm for finding the *greatest common divisor* of two numbers a and b. Euclid devoted part of his life to the prime numbers and indivisibility topics that belong unambiguously to the Theory of Numbers and are basic results. He also gave the first proof of *infinitude of primes*.

1.2. Some basic primality results

(i) If the unit place of a number is either 0, 2, 4, 6 or 8, it is divisible 2;

(ii) If the first two digits (counted from right) are divisible by 4, the whole number is divisible 4;

(iii) More generally if first k digits number of a given number is divisible by 2^k or 5^k the whole number is divisible by 2^k or 5^k;

(iv) If sum of digits is divisible by 3 or 9, number is divisible by 3 or 9;

(v) If difference of sum of odd digits and sum of even digits is divisible by 11 the whole number is divisible by 11;

(vi) Divisibility by 7 can be checked by taking modulus of 1001. For example,

$$598236779 \equiv 598 - 236 + 779 = 1141 \equiv 0 \pmod 7.$$

Hence 598236779 is divisible by 7.

§ 2. Pierre de Fermat (1601 – 65 A.D.)

The celebrated mathematician never published his works, especially those on Number Theory. His works were found in his letters to mathematicians and private marginal notes. He wrote down nearly no proof in Number Theory. He also had no models in this area. He repeatedly made use of Mathematical Induction, introducing the method of *infinite descent*.

Fermat's first interest was in Perfect Numbers of the form $2^h (2^{h+1} - 1)$, where $2^{h+1} - 1$ is a prime. The largest known perfect number is

$$2^{57885160}. (2^{57885161} - 1),$$

which is a 34 million digits long number. Till now no odd perfect number is known. No proof is found in known correspondences made during 1636 A.D. onward. From 1643 A.D., his interests shifted to Diophantine Problems.

Fermat defined some numbers of the form, called Fermat's numbers:

$$F_n = 2^{(2^n)} + 1.$$

Particularly,

$$F_o = 3, \quad F_1 = 5, \quad F_2 = 17, \quad F_3 = 257,$$

$$F_4 = 65537, \quad F_5 = 4294967297.$$

He conjectured that these numbers are primes. In 1732 A.D., Euler proved that F_5 is composite and factorized it as

$$F_5 = 641 \times 6700417.$$

In 1880 A.D., Landry proved that F_6 is also not prime. It is factorized as

$$F_6 = 274177 \times 67280421310721.$$

No prime has been found beyond F_4.

Fermat's numbers satisfy the recurrence relations:

(i) For $n \geq 1$, $F_n = (F_{n-1} - 1)^2 + 1$;

(ii) For $n \geq 2$, $F_n = (F_{n-1})^2 - 2 (F_{n-2} - 1)^2$;

(iii) $F_{n-1} = 2^{(2^n)}. F_o \ldots F_{n-2}$;

(iv) $F_n = 7 \pmod{10}$, for all $n \geq 2$.

Remark. It is very likely that Fermat's Primes are finite but this requires proof.

2.1. Another big challenge in the theory of numbers / Computer Science

Considering the above statement to be true then Fermat' Numbers are composite and have the divisor of the form $k \cdot 2^{n+2} + 1$, where k is some positive integer.

The numbers of the form $2^p - 1$ (called *Mersenne primes*) are primes for

p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, ….

The number $2^{21701} - 1$ is a prime having 6533 digits.

On 18th February 2005, the German medical doctor (Dr Martin Nowak) discovered $2^{25964951} - 1$, having 7816230 digits, as the largest prime number accounted till then.

Next, in March 2007, $2^{32582657} - 1$ was known as the largest known prime number having 98,08,358 digits. Assuming a page containing 60 lines each with 8 characters about 2000 pages were needed to print it. Thereafter, the Electronic Frontier Foundation (U.S.A.) offered $100,000 USD prize to discover still the larger prime. It was found by Dr Cooper, Department of Computer Science, University of Missouri that $2^{57885161} - 1$ is a prime of 1,74,25,170 digits. It is the largest known prime till now worked out at Global Gross Root Super Computer performing 150 trillion operations per second.

Fermat biggest achievements are his theorem (of 1640 A.D.):

$$\text{If } p \text{ is a prime then } a^{p-1} \equiv 1 \pmod{p},$$

for any a relatively prime to p, i.e. $(a, p) = 1$; but its converse is false, i.e. If $a^{n-1} \equiv 1 \pmod{n}$, $(a, n) = 1$, then n need not be prime.

Example. $2^{560} \equiv 1 \pmod{561}$, while 561 is not a prime.

Hence, Fermat Theorem does not guarantee for a number n to be a prime.

2.2. In 1640 A.D., Fermat made two very important statements:

(i) If $(a, b) = 1$, then $a^2 + b^2$ is not divisible by any prime congruent to -1 (modulo 4);

(ii) but it is divisible by any prime congruent to 1 (modulo 4).

Fermat also made a conjecture in 1637 A.D. (known as Fermat's last Theorem):

$$x^n + y^n = z^n \text{ ; has no integral solution if } n \geq 3.$$

He claimed (on the last page of his book) to have discovered truly marvellous proof of his above theorem but did not detail it because of the narrow margin therein. Did Fermat really know it or did he just mislead the mathematical community? Some mathematicians claim that Fermat never knew it. The unsolved problem had been the most notable theorem in the history of mathematics. It simulated the development of algebraic number theory in the 19th century and its proof in the 20th century. Prior to its proof, it was classified as an open problem in the "Guinness World Records". The reason was the largest number of unsuccessful proofs. Proof of Fermat's last theorem has been, finally, settled down after 358 years of hard work and continued efforts of mathematicians. Ultimately, Sir Andrew John Wiles, FRS (born on 11th April 1953), a British Mathematician and Royal Society Research Professor at the University of Oxford, a specialist in Number Theory proved it. His proof is most notable for which he received the 2016 *Abel Prize*.

§ 3. Computation in Number Theory algorithm

An algorithm is a specific procedure for solving a well-defined computational problem. Such methods are as old as any recognizable mathematics. The oldest algorithm is *Euclidean algorithm* for finding the *greatest common divisor* of two numbers. In the 5^{th} and 6^{th} century A.D. an algorithm called *kuttaka* (pulverizer), i.e. finding the quantities whose existence is ensured with proof or correctness.

There arise two questions:

(i) Can we compute it, (ii) Can we compute it rapidly.

For example, can anyone test whether a number is prime or not if it is not split into two prime factors? Doing so rapidly matters that there are fast methods for testing primality but truly no fast algorithm.

However, on a different side, something may not be computable at all. For example, there is no Turing machine, which can solve all Diophantine Equations.

In view of the above considerations, *factorization* and *primality testing* are two important concepts in Computer Science / Mathematics. From a purely academic motivation, it is an intriguing question to ask.

How can we determine whether a given number is a prime or not? The next logical question is if a number is composite can we calculate its factors? These two questions are invariably related. If we can factorize a number, obviously it is not a prime. If we cannot, then it is a prime. Hence, the definition of primality is very much derivable from *divisibility* or *factorability*. When we program through known and developed primality test and factorization algorithm, it will be clear that primality and factorization are intertwined. They occupy two levels of computational difficulty.

The study and development of a *test for primality* have been focused on the fast and deterministic algorithm. While primality and factorization are studied in many branches of mathematics, much of the recent study has been sparred by advances in cryptography and Number Theory. This application has motivated the recent interest in efficient primality and factorization problems. Primality and Factorization theory has a greater impact on modern encryption methods and public-key systems. The computational complexity of these two problems is different. Primality is an efficient problem as 20^{th} Century saw the biggest revelation in the area of cryptography when asymmetric encryption systems were conceived independently in 1976 by Diffie and Hellman. It was considered that the public key cryptosystem would completely outdate the *symmetric system* but it was turned out to be a false perception. This has weaknesses and computing power of computer was increasing day by day.

3.1. RSA algorithm: In 1977, Ron *Rivest*, Adi *Shamir* and Leonard *Adleman* gave RSA algorithm based on the product of two large primes. The algorithm was assumed to be secure. Martin Gardener wrote an article 'A new kind of cipher that would take millions of years to break' in his *Mathematical Game* column in *Scientific American*. This is how cryptography works. Gardener issued a challenge to his readers. He printed a ciphertext and also provided the public key that had been used to encrypt it.

The chosen number was

N = 114 381 625 757 888 867 669 235 779 976 146 612 010 218 296 721 242 362 562 561 842 935 706 935 245 733 897 830 597 123 563 958 705 058 989 075 147 599 290 026 879 543 541.

The challenge was to factorize N into two prime numbers p and q and use them to find the plain text. The prize was USD 100. Gardener' challenge took seventeen (17 years) before the cipher would be broken. On April 26, 1994, a team of 600 volunteers announced the factors of

$$N = p\,q,$$

where

$$p = 32\ 769\ 132\ 993\ 266\ 709\ 549\ 961\ 988\ 190\ 834\ 461\ 413\ 177\ 642\ 967\ 992\ 942\ 539\ 798\ 288\ 533,$$

and

$$q = 3\ 490\ 529\ 510\ 847\ 650\ 949\ 147\ 849\ 619\ 903\ 898\ 133\ 417\ 764\ \ 638\ 493\ 387\ 843\ 990\ 820\ 577.$$

Using these values as private keys they were able to decipher the message which was a series of numbers. When converted into text it yielded the magic words "Squeamish Ossifrage", i.e. a large number.

3.2. Another RSA challenge was the factorization of 155 digits number. A group of sixteen researchers, in four months, on a cluster of 292 computers factorized 155 digits number into primes p and q each of 78 digits:

p = 102 639 592 829 741 105 772 054 196 573 991 675 900 716 567 808 038 066 803 341 933 521 790 711 307 779,

q = 106 603 488 380 168 454 820 927 220 360 012 878 679 207 958 575 989 291 522 270 608 237 193 062 808 643.

There was another USD 10,000 bounty on factorizing 174 digit number

N = 188 198 812 920 607 963 838 697 239 461 650 439 807 163 563 379 417 382 700 763 356 422 988 859 715 234 665 485 319

060 606 504 743 045 317 388 011 303 396 716 199 692 321 205 734 031 879 550 656 996 221 305 168 759 307 650 257 059.

This number is known as RSA 576 for having 576 digits - when written in binary form. This number was factorized at German Federal Agency for Information Technology Security in 2003. The two prime factors p and q are

$$p = 398\ 075\ 086\ 424\ 064\ 937\ 397\ 125\ 500\ 550\ 386\ 491\ 199\ 064\ 362\ 342\ 526\ 708\ 406\ 385\ 189\ 575\ 946\ 388\ 957\ 261\ 768\ 583\ 317,$$

and

$$q = 472\ 772\ 146\ 107\ 435\ 302\ 536\ 223\ 071\ 973\ 048\ 224\ 632\ 914\ 695\ 302\ 097\ 116\ 459\ 852\ 171\ 130\ 520\ 711\ 256\ 363\ 590\ 397\ 527.$$

On 2nd November 2005, RSA 640 with 193 digits carrying a challenge of USD 20,000, was factorized by F. Bahr, M. Boehm, J. Franke, T. Kleinjun. It took five months. The number is

$$N = 310\ 741\ 824\ 049\ 004\ 372\ 135\ 075\ 003\ 588\ 856\ 793\ 003\ 734\ 602\ 284\ 272\ 754\ 572\ 016\ 194\ 882\ 320\ 644\ 051\ 808\ 150\ 455\ 634\ 682\ 967\ 172\ 328\ 678\ 243\ 791\ 627\ 283\ 803\ 341\ 547\ 107\ 310\ 850\ 191\ 954\ 852\ 900\ 733\ 772\ 482\ 278\ 352\ 574\ 238\ 645\ 401\ 469\ 173\ 660\ 247\ 765\ 234\ 660\ 9.$$

One of the prime factors of the above number is

$$p = 163\ 473\ 364\ 580\ 925\ 384\ 844\ 313\ 388\ 386\ 509\ 085\ 984\ 178\ 367\ 003\ 309\ 231\ 218\ 111\ 085\ 238\ 933\ 310\ 010\ 450\ 815\ 121\ 211\ 816\ 751\ 157\ 9.$$

The above team also factorized 663 bits RSA challenge integer with 207 digits. The factorizations were done by algorithm number field sieve based on Elliptic Curve Factorization.

Remark. Smallest currently open challenge is RSA 704 worth bounty $30,000. It has 212 decimal digits and is definitely composite.

$$N = 740\ 375\ 634\ 795\ 617\ 128\ 280\ 467\ 960\ 974\ 295\ 731\ 425\ 931\ 888\ 892\ 312\ 890\ 849\ 362\ 326\ 389\ 727\ 650\ 340\ 282\ 662\ 768\ 919\ 964\ 196\ 251\ 178\ 439\ 958\ 943\ 305\ 021\ 275\ 853\ 701\ 189\ 680\ 982\ 867\ 331\ 732\ 731\ 089\ 309\ 005\ 525\ 051\ 168\ 770\ 632\ 990\ 723\ 963\ 807\ 867\ 100\ 860\ 969\ 625\ 379\ 346\ 505\ 637\ 963\ 59.$$

§ 4. AKS *primality* test

On August 6, 2002, Computer Scientist Prof. Manindra Agrawal of I.I.T., Kanpur (India) and his students Neeraj Kayal and Nitin Saxena achieved a remarkable result. They discovered an efficient and deterministic test for the primality of a number. The mathematical community is

fascinated by this result not only because it answers an open question of long-standing but also for the mathematics behind it is beautiful and easily accessible as compared to other modern research advances.

4.1. Primality over the fields $K(i)$, $K(\rho)$, $K(\sqrt{m})$ and modular arithmetic over Elliptic Curves are new areas of research leading to its vast application to communication theory. On the other hand, computation of primitive irreducible polynomials of degree > 33 and study of non-linear logic are burning issues of the secure communication of modern era to face the ongoing problems to be faced by the country.

Now we have the following challenges for computer scientists to work on the following problems, if we are thinking of up-gradation in technological advances in the country:

Challenge 1. To work on the problems concerning primes and their applications leading to modern cryptography.

Challenge 2. Users of RSA should pick up such a large value of *n* to secure important information that it may be impossible to break the encoded message on this planet even if a large number of supercomputers may be working on it. So is the case with the discrete *log* problem.

Because of the Application of Algorithm Number Field Sieve based on Elliptic Curve Factorization, factorization is not difficult as it was a couple of decades ago. That is why public-key systems are in use in the commercial Sectors of the Government as well as in Private Sectors. These are also used for authentication and recognition purposes in the public as well as private sectors.

Conclusively, this paper raises the question of whether our younger people working in this area may take up such challenges and utilize their knowledge in the development of indigenous cryptosystems so that academia may help the Government/PSUs in Secure Communications.

References

Cindy, T. (2010). *Fermat's Numbers*. University of Washington Math 414 Number Theory Notes.

Hardy, G. H. & Wright, E. M. (2008). *An introduction to the theory of numbers.* (7 ed.). David Burton Paperback.

Misra, R. B. (2010). *Basic mathematics at a glance*. Saarbrücken, Germany: Lambert Academic Publishers. ISBN 978-3-8433-8696-8.

Singh, S. (1999). *The code book: The science of secrecy from ancient Egypt to quantum cryptography.* Fourth Estate and Doubleday.

Stein, W. (2008). *Elementary number theory, a computational approach.* Springer.

Acknowledgement

The second author wishes to express his sincere thanks to the referee for his valuable comments and the editor of the journal for bringing such useful information to its inquisitive readers stimulating their interest to explore the research in this domain. The paper is dedicated to the memory of Prof. Peter K. Anderson who had been a senior colleague of the second author at the Divine Word University, Madang (PNG).

Authors

(Late) Dr. S. P. Khare
Ex Director, Joint Cipher Bureau, Govt. of India, New Delhi;

Prof. Dr. Ram Bilas Misra
Ex Vice-Chancellor, Avadh University, Ayodhya / *Faizabad* (*India*); *Research and Strategic Studies Centre, Lebanese French University, Erbil, KRG* (*Iraq*); 380 A, Gomti Nagar Extension, Sector 1, Lucknow – 226010, U.P. (India);
Email: rambilas.misra@gmail.com; misrarb1@rediffmail.com

A Benford primer

Rik King

Abstract

This paper aims to excite the curiosity of the reader to achieve a basic level of understanding of the meaning of Benford's law. Benford's law concerns the prevalence of first and subsequent digits which appear in naturally occurring numerical transactions. One possible forensic application is in the detection of fraud in machine-generated sets of data, which do not obey this law. In addition to explanation with illustrative examples, some programs constructed in Excel and R are provided.

Keywords: Benford, Excel, first digit, fraud, primer, R, scale invariance, second digit.

Introduction

It is thought that Mathematics dates from the Sumerian civilization circa 3000 BC., from which many centuries of mathematical research displaying sophistication and complexity follow. So there is something surprising and very wonderful about the simplicity of a discovery depending merely on the numbers 0,…,9. This has popped up in the modern era; and with no insignificant mathematical curiosity, it turns out to be the focus of a very intense, expanding current research area.

This short article aims to excite the curiosity of the reader to achieve a basic level of understanding of the meaning of Benford's law. In addition to explanation with examples, some programs constructed in Excel and R are meant to be worked through in a 'learn as you go' style.

The leading digit

To begin with, think about the numbers which form so many of our transactions in everyday life. Usually, these are greater than 0, that is, positive. Now, each number has what is called a leading digit, which is just the first part of the number on its own. So, for example, the leading digits of the numbers 7.62, 34 and 0.0528 are 7, 3 and 5 respectively. A leading digit can be any one of 1, . . . ,9 (0 is not in the list because then the number would be 0, and not of any interest). Presented with a large file of numbers to be examined, intuition would suggest all the leading digits i.e 1 to 9, should appear as frequently as each other - as many ones as twos as threes etc. as nines - indeed, why should it be otherwise? In Statistics, however, intuition often misleads, which is how it turns out in this case.

The surprising fact is, that, in very many collections of numbers, there are fixed proportions of leading digits 1 to 9, and these proportions are far from equal. The values are given by Benford's Law (Benford, F.,1938), a statement of which is as follows:

The proportions for d being the first digit of a number are given approximately by

$$\log_{10}\left(1 + \frac{1}{1+d}\right) \qquad (1)$$

where d is any of (1,…,9). This gives rise to Table 1 below:

Table 1: Benford's Law

Digit	1	2	3	4	5	6	7	8	9
Proportion	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

So, this means that in a collection of numbers there should be about 30% ones, 17% twos - just under 50% ones and twos combined, but only 5% nines. Figure 1 displays these proportions.

Thus, it turns out that the first digits of numbers in many collections of data, rather than following a uniform distribution (equal amounts), follow a discrete logarithmic distribution, unusual, because it is not described with the help of any parameter: the formula involves only d, the digit number, and no other variable. The logarithmic term may be to any base, but base 10 is most frequently used.

Frank Benford was a physicist and the connection between his work and that of an earlier (1881) researcher Simon Newcomb, who was an astronomer, is truly fascinating. Stoessiger (2013) gives an interesting account of the link between the work of the two researchers.

Exploring a data file

The working for this section follows the method of Lanham (2019), which details how to obtain a large data source suitable for investigation. For every state in the USA, there is a State Occupational Employment and Wage Estimates (SOEWE) document, which provides information on employment and wage estimates for various occupations, with data collected directly from employers in all industry sectors. Data sets for different years are downloadable from the US Bureau of Labor Statistics[1].

This section is best read by working step-by-step through a data file, as described below after downloading into Excel a data set for any year from SOEWE. Here we discuss the SOEWE (2019) data set comprised originally of 36,383 records, reduced to 34,853 entries after sorting and the removal of incomplete lines.

Some simple Excel commands have been used to extract the first digit for each of the 34,853 records, and then collect the digits into groups and count the number in each group. The cleaned example file SOEWE.xls displays the following columns:

(i) Column C: the employment numbers

[1] *https : //www.bls.gov/oes/current/oessrcst.htm*

(ii) Column D, the first digit of each number in column C, extracted by the Excel function LEFT(.)

(iii) Columns F - N, the count of digits in column D, extracted by the Excel function COUNTIF(.,.)

The final proportions for digits 1 to 9 do not appear on the spreadsheet but should be calculated by the reader and checked from Table 2 and Figure 1 below:

Table 2: SOEWE result

Digit	1	2	3	4	5	6	7	8	9
Proportion	0.298	0.167	0.123	0.101	0.083	0.069	0.058	0.054	0.044

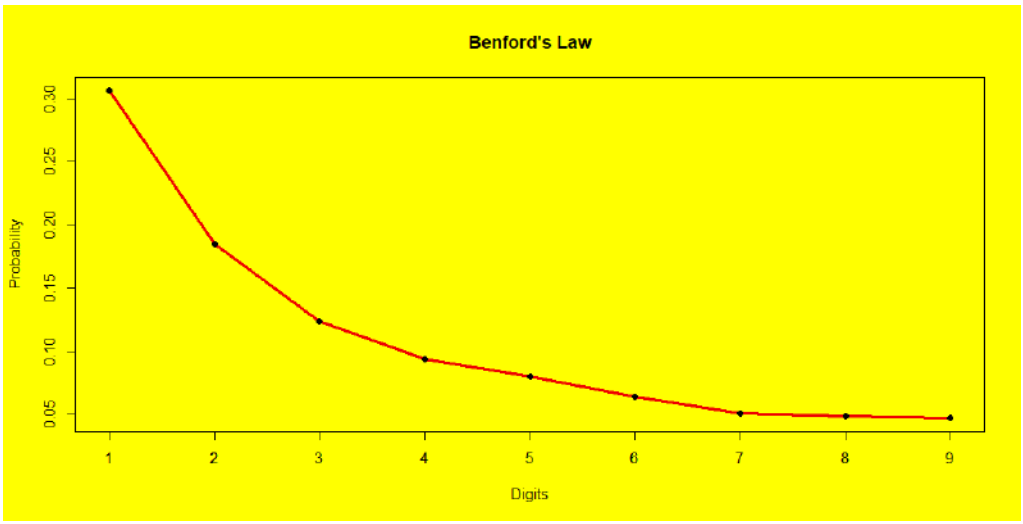


Figure 1: Benford's Law.

In Figure 2 results are compared with the probabilities noted by Benford, showing good agreement (helped along by the large size of the SOEWE file).

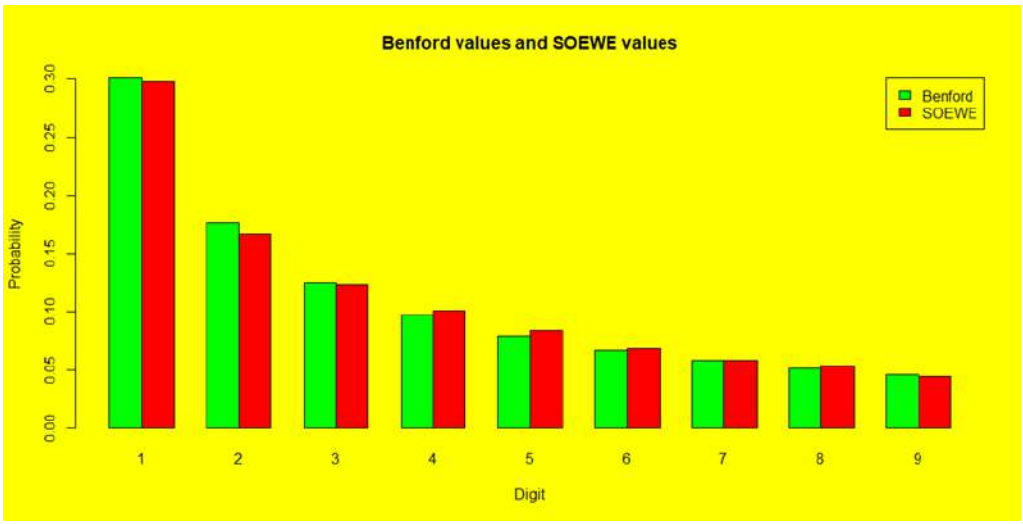


Figure 2: Benford values and SOEWE values.

Using R

While Excel spreadsheet is a useful method for processing large data files, the same results may be achieved through longer processes by using R. There is a sophisticated R package entitled BenfordAnalysis.R due to Cinelli (2018) which downloads data from several formats and conducts analysis at elementary and higher levels.

Fibonacci numbers

It is known that many mathematical sequences obey Benford's Law and amongst these is the well-known sequence due to Fibonacci where numbers are defined by the recurrence relation $F_{n+2} = F_{n+1} + F_n$, with $F_0 = 0$ and $F_1 = 1$. However, from the demonstration point of view, it is unfortunate that the terms grow large very quickly, and even using an efficient recursion, generating a large data file is beyond the capacity of an average laptop. A bypass of this difficulty is provided by Binet's approximation (Miller, 2015), which is given by:

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n \qquad (2)$$

Since, however, terms with n large are to be generated, it is useful to simplify the above to:

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n \qquad (3)$$

This second form of Binet's approximation is employed in the program Fib.R, listed in Appendix (i). There, 1000 Fibonacci numbers are generated and the leading digit extracted; a count is made of the digits 1 to 9 and their proportions of the total (1000) calculated. The results, showing good agreement with the Benford values of Table 1, are displayed in Table 3.

Table 3: Benford's Law & Fibonacci numbers

Digit	1	2	3	4	5	6	7	8	9
Benford	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
Fibonacci	0.301	0.177	0.125	0.096	0.080	0.067	0.056	0.053	0.045

The Fibonacci numbers are discrete - i.e. in distinct units. It is useful now to look at a different growth process, a continuous set of numbers, this time drawn from Finance.

Finance

The material of this section follows the method of Miller (2015), which begins with the well-known formula for the amount \$A, accruing from an amount of \$P invested for n years at a rate r% per annum:

$$A = P(1 + 0.0r)^n \qquad (4)$$

An adaptation of the above formula is useful for considering the leading digits of the amounts. If d is the leading digit of an amount invested, after n years, the amount grows and the leading digit d moves up by 1 to become (d + 1). Thus, amount $d(1 + 0.0r)^n = (d + 1)$. Solving for the number of years n is more appropriately done with logarithms to the base 10, rather than the usual natural logarithms, since the digits 1...9 are involved:

$$n = \frac{\log_{10}(\frac{d+1}{d})}{\log_{10}(1.0r)} \qquad (5)$$

From the foregoing format, it is possible to answer the following question: how long does it take for a deposit of $1 to grow to over $2, then how long for over $2 to become over $3 and so on? Notice that there will be intervals during which the leading digits of the amounts will be respectively 1, 2, 3,...,9. The quantities of interest are the lengths of the intervals during which the leading digit stays the same. This is because any count of digit frequencies will show a higher reading for a leading digit, which persists for a longer interval.

Listed in Appendix (ii) is the program My Deposit.R. It accepts the arguments amount and interest rate (no % sign) compounded annually, time being taken as indefinitely long. The function first needs to be defined in R, after which it is ready to be run for any specific example.

The following paragraph uses the results from that program, which the reader is encouraged to run. Consider the results from My Deposit (10, 5) which describes $10 which has been invested at 5% per annum.

The output appears in two sections. The first header viz. ”time to next digit up” indicates in years, how long it takes for the $10 to become $20, then $20 to become $30, then $30 to become $40, all the way up to $90: in other words, how long an amount remains with the same leading digit before jumping up to the next digit. So, reading from the output shows that it takes 14.2 years for $10 to become $20 and 8.3 years for $20 to become $30, etc. Therefore there were many more leading digit ones than twos, because of the relative lengths of the time intervals in which the leading digit did not jump up to the next value. The times for a digit to “jump up” steadily decrease. The second part of the output with the header time as a fraction of total time: 1 - 9 lists each time spent with a leading digit as a fraction of the overall time for the money to get from $10 to $90. For the example above, going from $10 to $20 took 0.301 of the total time, but from $80 to $90 only 0.045.

The program should be run several times for varying initial amounts when it becomes apparent that the second part of the output - a fraction of total time - is always the same. This is because it is the ratios of quantities (due to the interest rate) that count, and not the quantities themselves: it takes as long to get from $100 to $200 as from $10 to $20. Also, running the program for varying interest rates results always in the same fractional times. The core of the calculation - what is actually happening in the program - is exhibited in Table 4 below: The right-hand column, is, of course, just the Benford numbers.

Table 4: Program results

Digit	Log - Log	Result
1	log 2 - log 1	0.301
2	log 3 - log 2	0.176
3	log 4 - log 3	0.125
4	log 5 - log 4	0.097
5	log 6 - log 5	0.079
6	log 7 - log 6	0.067
7	log 8 - log 7	0.058
8	log 9 - log 8	0.051
9	log 10 - log 9	0.046

The above example is highly simplified - so, in particular, invested amounts are often not constant and interest rates typically vary over time, however, for the sake of simplicity in the demonstration, these complicating factors have been ignored.

There is a further very important point connected with My Deposit.R. It worked for dollar units of investment, but it could have worked just as well for an investment of the same amount converted to francs or yen - the final results would be connected by the currency conversion factors. The behaviour depended not on quantities, but on the ratios of quantities, governed by the interest rate: this is a point amplified in a later discussion on scaling.

Applications

Benford's law is not universal. The data must show a full order of values from 1 to 9 fold, and must not have imposed natural maxima or minima e.g. the petal sizes of a particular species of flower would not be suitable data. It does not apply to manufactured numbers such as car license plates, telephone numbers or bank account numbers. In general, it holds for natural numbers such as the area of landmasses, and volumes of river flow; and for many fundamental physical constants and quantities from the natural sciences.

A big area of application is finance (the topic of the final section). Various types of data certainly follow the law very accurately: these include stocks, shares, mathematical combination of numbers, such as quantity multiplied by price disbursements, and sales numbers.

A cautionary remark, however about general criteria for Benford suitability is in order. The great expansion in the diversity of applications, never envisaged even a few short years ago e.g. intensities in digital imaging, have consigned some previously held certainties into the hypothesis category, so now some writers e.g. Miller (2019) refer to previously held certainties on applicability as hypotheses, so e.g. the 'spread hypothesis'. There are now, however, formal approaches to proving whether a system satisfies Benford's Law. (cf. Berger & Hill, 2011).

Their work also provides the answer to the following question which may cross the mind of the reader: Is it possible to manufacture a set of numbers, which with certainty, will display the values of the Benford distribution? The short answer is 'yes'.

To see that this is so, run the program Benford.R to be found in Appendix (iii). It simulates 5000000 numbers; a typical output is shown in Table 5 below:

Table 5: Benford's Law & simulated numbers

Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Benford	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
Simulated Nos	0.301	0.176	0.126	0.097	0.078	0.067	0.058	0.051	0.046

The following section may be skipped on first reading but for the more curious reader, it gives details of the standard inverse transform method for generating a random variate for the case of a logarithmic distribution.

Generating Benford numbers

In equation (1), a working definition of Benford's law, it is implicit that the proportions may be expressed as probabilities. Thus, the equation may be recast as:

$$P(D = d) = \log_{10}\left(1 + \frac{1}{1+d}\right) \qquad (6)$$

where D is the probability that any digit will be d, d E(1,9). Actually, it is easy to show that equation (6) above holds for any base, not just 10, but in the interests of simplicity, base 10 will continue to be used here.

Equation (6) sums to 1 as a true probability distribution must do: the demonstration of this proceeds as follows:

The probability that the first digit D = d is given by

$$P(D = d) = \sum_{d=1}^{10-1} \log_{10}(\frac{d+1}{d}) = \log_{10} \prod_{d=1}^{9} \frac{d+1}{d} \qquad (7)$$

$$\log_{10} \prod_{d=1}^{9} \frac{d+1}{d} = \log_{10} \frac{(9+1)!}{(10-1)!} = 1 \qquad (8)$$

which confirms equation (6) as a probability density function. Knowing the probability density function (pdf) makes it possible to get the cumulative distribution function (cdf), which is obtained as follows:

$$P(D \leq d) = \sum_{1 \leq d' \leq d} P(D = d') = \sum_{1 \leq d' \leq d} \log_{10} \frac{d'+1}{d'} = \log_{10}\left(\prod_{1 \leq d' \leq d} (\frac{d'+1}{d'})\right) \qquad (9)$$

and then

$$\log_{10}\left(\prod_{1\le d'\le d}\left(\frac{d'+1}{d'}\right)\right)=\log_{10}\left(\frac{2}{1}\cdot\frac{3}{2}\cdots\frac{d+1}{d}\right)=\log_{10}\left(\frac{(d+1)!}{d!}\right)=\log_{10}(d+1) \tag{10}$$

so that the distribution function is

$$F(x)=P(X\le x)=\log_{10}(d+1) \tag{11}$$

for $x = 1, 2, \ldots, 9$

Then the usual inverse transform method given for a Benford variate X is

$$X=\lceil 10^{U}-1\rceil \tag{12}$$

whence X ← $\lfloor 10^{U} \rfloor$; this is the procedure implemented in Benford.R from Appendix (iii).

Scaling

A special feature of data sets that obey Benford's law is the following: multiplying a set of numbers that obey Benford's law by some constant number will produce another set of numbers that also obeys Benford's Law. Whatever the constant, the new, different, numbers will also obey the law.

Consider the following small example. Suppose a set of numbers is (1.3, 4.5, 6.2, 2.4, 8.0, 3.0) with leading digits in bold type. Multiplying the set by, say, 1.5, gives a new set. Using a calculator will show that the new leading digits are (1, 6, 9, 3, 1, 4). The numbers have changed, some leading digits disappear, others reappear; so in a large set, it is plausible that the proportions of leading digits might remain the same.

For a demonstration, run the program Fib.R, choosing again to calculate 1000 numbers. This time a small positive constant (= 1) is to be entered; it is to multiply (scale) the Fibonacci numbers. So, for example, the input might be Fib(1000, 1.5). The output, after making allowance for round off error, is almost exactly the same as in the Table 3 Benford values.

This unchangeable feature of Benford's Law is described as scalability or scale invariance. While the above program is only a demonstration, Berger & Hill (2011) gave a formal proof of this characteristic based on manipulations in a σ algebra (but that is beyond the scope of this primer). Further of interest is this fact: Pinkham (1961) had already shown that there can be no other scale invariant distribution of first digits: that is, Benford's is the only one.

A consequence of the scaling property is that physical measurements of natural phenomena in any set of units will obey Benford's Law. So, for the previously mentioned data set of world river lengths, it doesn't matter whether the units are miles or kilometres: nature doesn't discriminate - scaling operates.

Number invention and tampering

One of the first in the field of applications to finance was Nigrini (1999), who, in 1993, was a young accountant. He gives details of an Arizona USA fraud trial in which he was involved. A company employee, in the course of his work, had paid random amounts totalling about $2,000,000 into his own bank accounts. Random was the problem - for the employee, of course; but not for the Court, which compared actual digit occurrences with Benford's Law, and brought down a finding of guilty of fraud. Nigrini's paper is aptly titled 'I've Got Your Number'!

How forensic finance investigation operates will now be illustrated by a basic level example. Table 6 below exhibits first digit data from a file of 2525 entries, containing falsified entries, as quoted in Nigrini (2008), and displayed in Figure 3, where there is an obvious divergence from the Benford values.

Table 6: False Data

Digit	1	2	3	4	5	6	7	8	9
Number	668	381	256	244	235	212	208	179	142

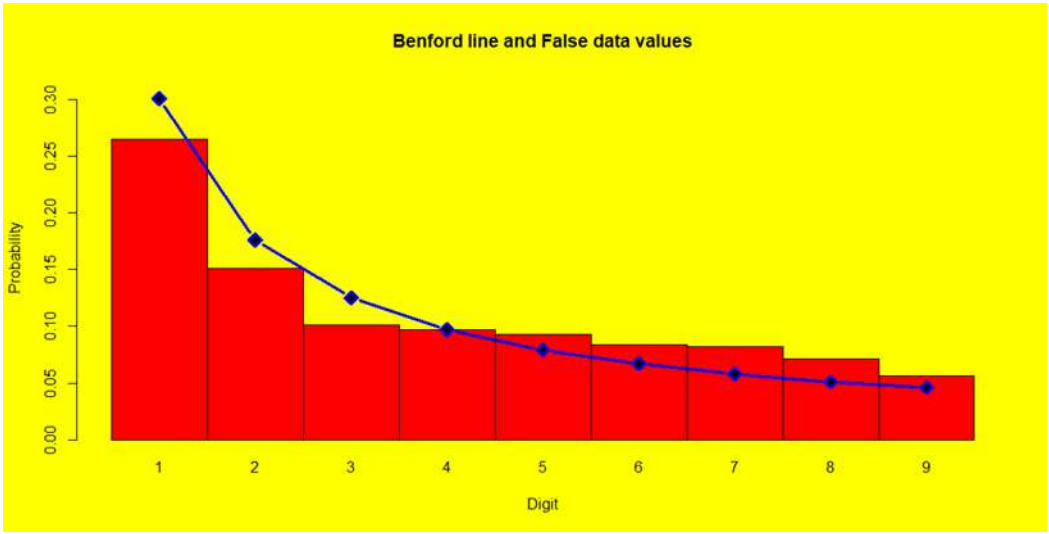


Figure 3: The Benford line and False data values.

The data in Figure 3 is an example only. For real-life data, ascertaining Benford compliance often depends on subsequent Z-tests and ChiSquare tests.

Further digits

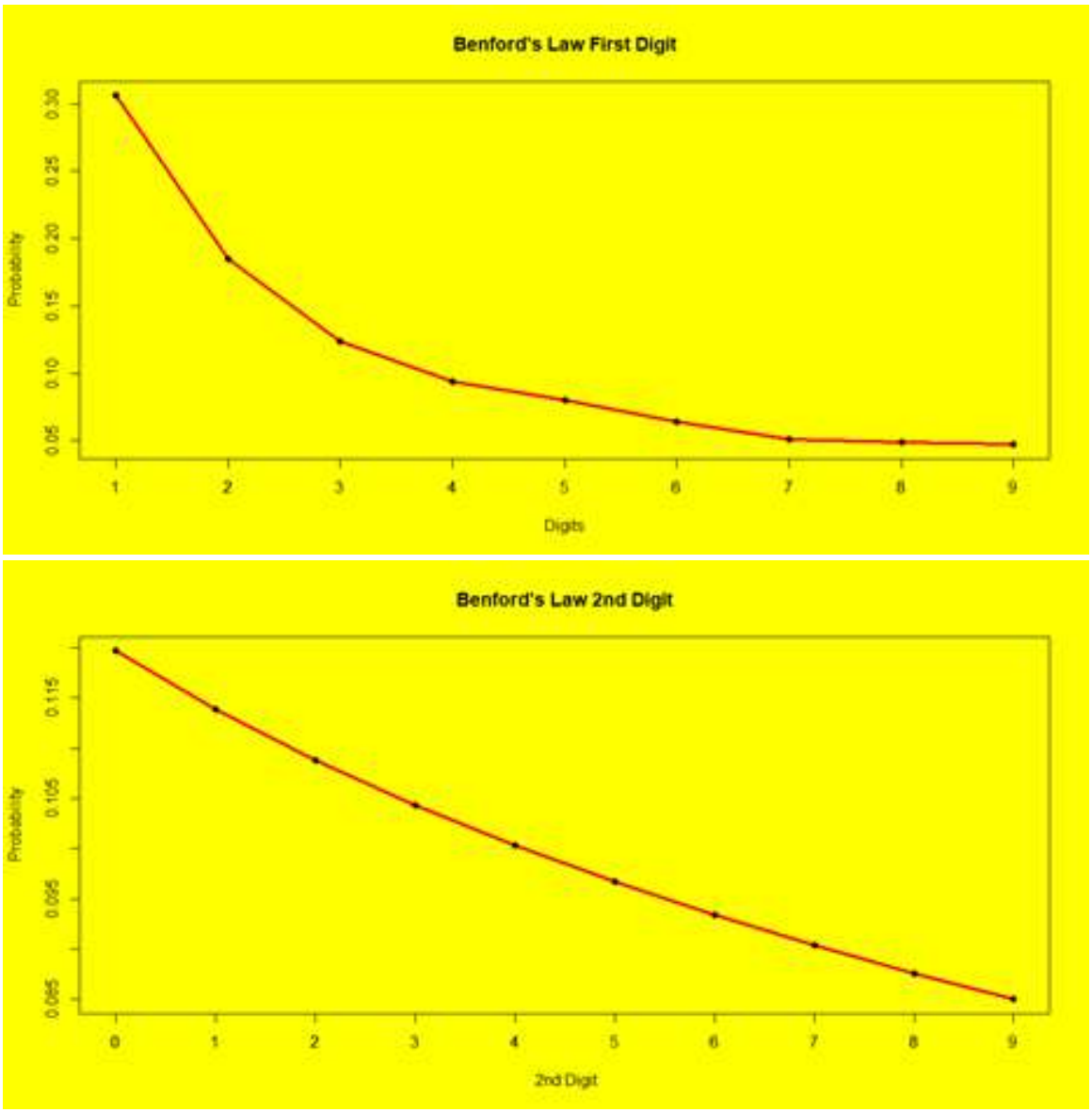
Benford probabilities apply not only to the first digits of numbers, but also to second, third, and fourth digits, with a probability law which is more complicated than the one used so far; but a log10 distribution still applies; additionally, there is a term of the $(1 + \frac{1}{1+d})$ type.

A little preliminary explanation may be useful. Suppose 4 to be a second digit: it can occur as 14, 24, 34, 44,…, 94 so the probability of a 4 occurring must involve a sum of probabilities;

and likewise for the other digits out of (1, . . . ,9). Also, there must be a 10 multiplying any second digit, to ensure that it is fixed in the second decimal place. Finally, 0, which was not admissible as a first digit, can occur as a second digit. Putting all this together results in the probabilities of occurrence of digits first and higher:

$$P(d_1, d_2, ...dm) = \log_{10}\left(1 + \frac{1}{\sum_{j=1}^{m} \frac{1}{10^{m-j} dj}}\right) \qquad (13)$$

where P is probability; d_1, d_2, ...,d_m, are digits with $d_j \geq 2$, and m is an integer $0 \rightarrow 9$.
The first digit place is where the distribution of Benford's Law differs the most from the uniform random distribution; subsequent digit probabilities tend more and more towards the uniform. The start of the 'becoming more uniform trend is demonstrated with the probabilities of the first and second digit in Figure 4(a), 4(b) below:



(a) First Digit (b) Second Digit

Figure 4: First and Second Digit

Amongst the many specialized applications of multiple digit analysis, two are mentioned briefly below.

(i) Finance

Among the different general levels of digit analysis, the test for the first digit is the most effective in pointing to suspicious data. It also plays a role in deciding on the size of a sample considered for further investigation, an important factor for auditing cost. Different tests have different functions: one set of five tests, following ACFE (2020), is as listed below: (a) The first digit test (b) The second digit test (c) The first and second digit test (d) The first three digits test (e) The last two digits test. It is impossible to mention here the huge spread of applications - basically any transaction, government or private where money may change hands.

(ii) Election fraud

The second digit test - 2BL for short - has played a role in detecting figures that may suggest election fraud. While an enormous amount has been written on this topic, involving specific election results from particular countries (see Mebane, Walter R.Jr. for examples), academic debate on the application of 2BL to elections is ongoing. The existence of uncertainty at that level of discussion is a clear signal for the conclusion of a primer level understanding of Benford.

Conclusion

The understandings gained by working through this primer should be adequate to progress on to one of the specialized areas of application of Benford's law. These are far too numerous to mention, but a few are physics, computer science, scientific data quality control, digital imaging forensics, and biology. Their number is constantly expanding.

Appendices

(i) Fib.R

Fibonacci nos. & Binet's Approximation. Fib(n, k = 1) generates the first n Fibonacci numbers and multiplies them by constant k (defaultvalue = 1)

```
Fib <- function(n, k = 1)
{ #------ begin function
  # fd extracts first digits
  fd <- function(x) {
    a = log10(x) %% 1; floor(10^a) }
  # nos holds generated numbers
  nos <- c() ; s = sqrt(5)
  for(i in 1:n){
```

```
   nos[i] =1/s * ((1 + s)/2)^i}
 ben = fd(k * nos) ;
 round(table(ben)/n, 3)
} #------- end function-
```

Examples
benf = Fib(1000) generates the first 1000 Fib.nos
benf = Fib(1000, 1.5) generates the first 1000 Fib.nos multiplied by 1.5

(ii) My Deposit.R
My Deposit.R takes the arguments amount and interest rate. It outputs times and fractional times for digit d to become d + 1

```
My_Deposit <- function(amount, rate){ # begin function
dep = c(rep(0,9)); R =log10(1 + rate/100)
for(d in 1:9){dep[d]= log10((d + 1)/d) *1/R}
dep = round(dep,3)
pc.time = round(dep/sum(dep),3)
out = list("time_to_next_digit_up" = dep,
"time_as_fraction _of_total_time: 1 -> 9" = pc.time)
return(out)
} # end function
```

Example: My_Deposit(10,5)

(iii) Benford.R
Benford.R generates numbers obeying Benford's Law

```
# N.B. increments are drawn from R's Uniform Distribution
# Benford.R
# Final version of Benford numbers
N = 500000; # number of replications
x = floor(10^runif(N)); # digits
benf = table(x)/N
round(benf,3)
```

Output: 500000 first digits sorted into bins (1,. . . , 9)

References

Berger A.& Hill T.P. (2011). A basic theory of Benford's Law. *Probability Surveys, 8,* 1-126.

Benford, F. (1938). The law of anomalous numbers. *Proceedings of the American Philosophical Society 78,* 551-572.

Cinelli, C. (2018). Benford Analysis.R, Retrieved 1 November 2021, from http://github.com/carloscinelli/benford.analysis.

Lanham S. W. (2019). Analyzing big data with Benford's Law. *American Journal of Business Education, 12*(2), 33-42.

Leemis L. (2018). *Probability*, Lightning Source: USA.

Miller S.J. (2015). *A quick introduction to Benford's Law*, Princeton University Press: USA.

Newcomb, R. (1881). Note on the frequency of use of the different digits in natural numbers *American Journal of Mathematics, 4*, 39–40.

Nigrini, M. (2008). The problem of false negative results in the use of digit analysis *Journal of Applied Business Research, 24*(1), 17-26.

Nigrini, M. (1999). I've got your number *Journal of Accountancy, 187*(5), 79-83.

Pinkham, R. S. (1961). Ann. Math. Stats. 32: 1223-1230 R, https://www.r-project.org

Stoessiger, R. (2013). Benford's Law and why the integers are not what we think they are: A critical numeracy of Benford's Law *Australian Senior Mathematics Journal, 27*(1), 29-46.

US Bureau of Labor Statistics (2021). *Occupational employment and wage statistics* Retrieved 1 November 2021, from https:/www.bls.gov/oes/current/oessrest.htm.