
Comparing spreadsheet and R platforms for teaching university mathematics

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Abstract

Mathematics uses a highly abstract set of symbols to represent its concepts and procedures. The accompanying verbal descriptions are not greatly less abstract and this makes the study of mathematics daunting for less courageous students. Readily available computer software can be leveraged to allow illustrations of mathematical techniques to real world situations using Modeling and graphical analysis. Our experience is that this can be a powerful motivating factor for university students to develop an appreciation of mathematics.

Key words: Excel, R, math problems, models, probability and statistics, mathematical Modeling.

Introduction

Mathematics can be considered from the perspective of a number of components. It takes an abstract form of symbol manipulation with little intuitive interpretation. The symbols represent abstract concepts, quantities or operations. The symbols also have an associated verbal language which in some cases are more intuitive than the symbols, but which also require learning. Then real world models, widely used in science and engineering research, can be found to which the symbolic process can be applied in a process which can give helpful meaning to the procedures represented by symbols. In the real world, a given mathematical procedure can be applied to a diverse range of models giving rise to the concept of the “Unifying power of mathematics” (Kreuzsig, 2011, p 69). Students can be motivated to learn and enjoy mathematics when learning and teaching moves beyond symbol manipulation and embraces verbal, graphical and Modeling approaches.

This paper seeks to focus on readily available software which can be used to model mathematical processes either in graphical form or in the form of application to working real world models. In particular it will consider spreadsheet software (e.g. Microsoft Excel) as a useful Modeling tool (Neuwirth & Arganbright, 2004) and the freely available R language and environment for “exploring, visualising and understanding data” (Kabacoff, 2011). Examples will be given to illustrate the way these tools can be used to illustrate the abstract world of mathematical symbolism and encourage its study.

Nature of mathematics

Mathematics uses symbolism to represent abstract concepts originating from the analysis of space and time by the ancient Greeks. Modern Mathematics, of course, can represent a more abstract analysis of concepts beyond those of space and time. It can analyse the truth of logical procedures, for example, using set theory. However, for the learning of mathematics there are the aspects of language and unintuitive symbolism to be dealt with. Herein lies the difficulty of learning mathematics since, at first sight, mathematics appears as a new language comprised of abstract symbols only. However, this need not be the end process. Symbols which represent procedures or functions can be expressed using a readily available spreadsheet technology. Although not primarily a Modeling tool, a spreadsheet can be used, in the first instance, as a means of Modeling. Also the readily available R platform, although primarily a data analysis tool, can be used for graphical display of mathematical processes.

Mathematical modeling

A mathematical model is here understood to be a set of symbols and procedures of manipulation which can mimic aspects of a real world situation. Also the model is a simplified version of the situation with the introduction of simplifying assumptions to remove complications found in real world situations. With the advent of readily available computer technology both students, designers and inventors can inexpensively test their projects using a model under variations of component parts without having to build physical models or test the actual project the model represents. For example to test if the vibrations from a genset (electrical generator) in the basement of a building matches one of the resonant frequencies of the building, mathematics models would clearly be preferable.

Using Excel as a modeling tool

Excel is a well-known spreadsheet technology used primarily for data recording and analysis. However its wide range of mathematical functions and graphical tools make it a useful tool to introduce students to mathematical modeling and to explore details of mathematical processes. The model is built from a sequence of mathematical functions dependent upon quantities inserted into input cells. As well as being a useful testing tool for real world situations, it enables the student to explore the details of the mathematical process being modelled. Input values can be varied and output can be viewed as a numerical quantity or variation in a graphical display. Some examples of a Modeling process can now be considered.

Modeling examples

Financial Model

Here we consider the case of investing money in a bank. If K50 was deposited as the initial deposit and the bank was to add 5% interest to that amount

annually, then to find the amount after 15 years using a spreadsheet, we begin by entering input data in cells (Figure 1).

	B	C	D	E	F
1	increase rate			0.15	15
2	initial money		K 50		
3					
4	Years	Interest	Balance		

Figure 1 Input data has been entered.

Having input the available data and created space for calculating years, interest and balance, we then proceed on with the calculating of years up to 15 (Figure 2).

Years
0
=B5+1
2
3
4

Figure 2 Numbers from 0 to 15 are added.

In the initial year there will not be any interest so the balance will be equal to the deposit but in the following years, the interest will be calculated as:

$$= \text{balance} * \text{interest rate.}$$

The new balance will be:

$$= \text{interest} + \text{previous balance.}$$

The calculation of interest and a new balance are now created (Figures 3 & 4).

increase rate	0.05	
initial money	K 50	
rs	Interest	Balance
		50.00
	=D5*\$E\$1	52.50
	2.63	55.13
	2.76	57.88

Fig 3: Interest is calculated.

Interest	Balance
	50.00
2.50	=D5+C6
2.63	55.13
2.76	57.88

Fig 4: A new the balance is calculated.

Excel's =FV (Future Value) function can be used instead of doing the above calculation to get the same result (Fig 5).

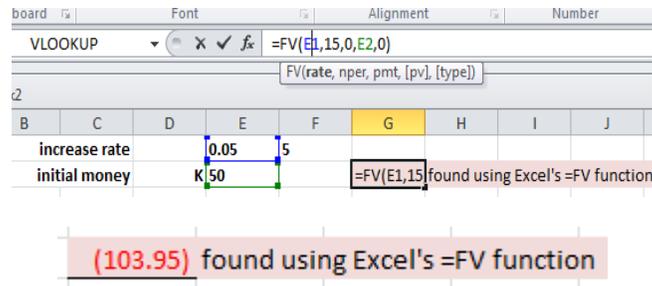


Figure 5 A short cut method finds the new balance after 15 years, using the =FV function.

A graph showing financial growth over time can now be readily constructed (Fig 6).

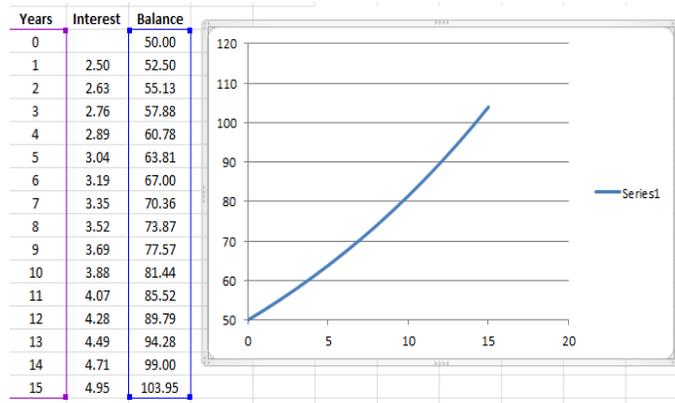


Figure 6 Financial growth with time is now presented.

Furthermore, to make the presentation more interactive, a scroll bar can be inserted which will be linked to the changing interest rate to show the variation in balance will if as interest rate is varied (Figure 7).

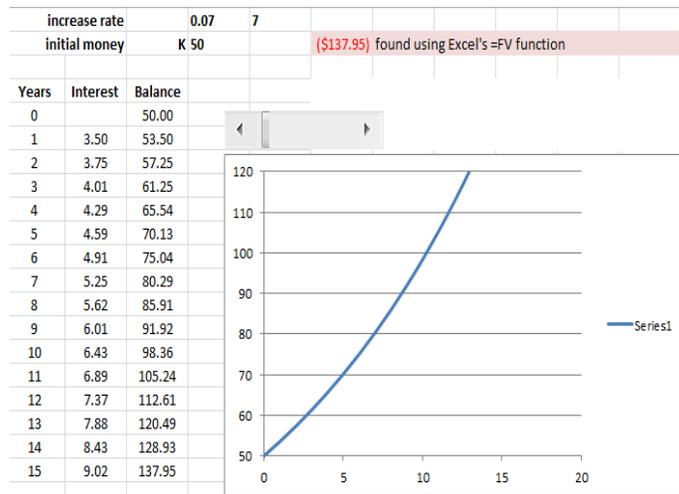


Figure 7 Scroll bar allows continuous variation of interest rate and the consequences of change can be explored. Figure 7 is repeated with interest rate of 5% being varied to 7%.

Using the scroll bar to increase the rate from 5% to 7% will then produce the interest at year 1 to be K3.50 and the new balance will be K53.50 (Figure 7). We now turn to a physical model to interpret a simplified naturally occurring situation.

Predator-Prey Model

Here we consider the well-known Predator-Prey Model (Neuwirth & Arganbright, 2004, p. 68) which attempts to model how two inter-dependent animal populations grow and decrease over time as they interact upon each other and we discuss the factors that might cause the changes.

This model is a study of predator (in this case fishermen) and prey (fish) in a certain fishing location. The predator and prey populations don't remain constant but can change in various ways. For example, when there is no fish catching, the fish population will grow at an exponential rate (assuming unlimited food supply) but with fish catching, the fish population will decrease proportionally to the size of the population of fishermen.

The mathematical representation of these assumptions on the size of the fish population is:

$$\text{ChangeNumberOfFish} = (\text{InitialGrowthRateOfFish} - \text{PreyFactor} * \text{Fishermen}) * \text{Fish}$$

where the "PreyFactor" is the fraction of the total number of available fish caught per fisherman and where the variables "Fish" and "Fishermen" refer to numbers in the corresponding populations. The equation implies that increases

in the population of fishermen will cause a reduction from the initial or natural growth rate of the fish population.

Similarly for the fishermen population we have the mathematical representation as:

$$\text{ChangeOfFishermenNumbers} = (-\text{InitialDiminishingRateOfFishermen} + \text{FeedingRate} * \text{Fish}) * \text{Fishermen}$$

where the feeding rate is the fraction of the total number of fish caught per fisherman. The equation implies that increase in population of fishermen is dependent on the size of the fish population. If there are many fish then more fishermen will be attracted to the fishing location.

Solving these two equations simultaneously will give the actual changes in the populations of fish and fishermen over time, given all these assumptions. The above equations for the model can be inserted into the spreadsheet model (Fig 8).

growth rate fish	0.04			
prey factor	0.0004			
dimin. rate fishermen	0.08			
feeding rate	0.0001			
	number		change	
period	fish	fishermen	fish	fishermen
1	1000	100		

Figure 8 Initial data for the model are entered.

Assuming there are 1000 fish and 100 fishermen in the first fishing period, we now calculate the change in numbers of fish and fishermen as shown using available mathematical functions (Fig 9).

1	growth rate fish	0.04		
2	prey factor	0.0004		
3	dimin. rate fishermen	0.08		
4	feeding rate	0.0001		
5				
6		number		change
7	period	fish	fishermen	fish
8	1	1000	100	=B8*(\$C\$1-\$C\$2*C8)

Figure 9 Changes in fish populaion are calculated.

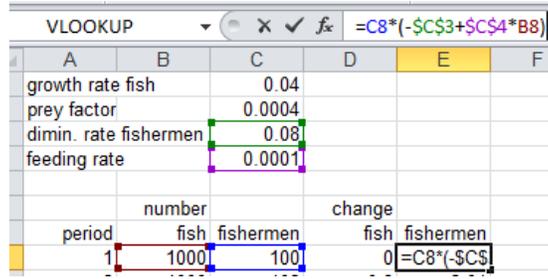


Figure 10 Changes in numbers of fishermen are calculated.

To obtain the number of fish and fishermen in a second period the calculations will be as follows:

$$\begin{aligned} \text{period} &= 1 + 1, \\ \text{fish population} &= 1000 + \text{change in size of fish population}, \\ \text{fishermen population} &= 100 + \text{change in size fishermen population}. \end{aligned}$$

From then on, the formulae will be the same for all the consecutive periods, so instead of repeating the calculations, the available drag down option can be used to calculate the new values (Fig 11).

period	number		change	
	fish	fishermen	fish	fishermen
1	1000	100	0	2
2	1000	102	-0.8	2.04
3	999.2	104.04	-1.61471	2.072477
4	997.5853	106.1125	-2.43909	2.096626
5	995.1462	108.2091	-3.2677	2.11166

Figure 11 Population changes over time are displayed in table form.

A 2 dimensional population graph of the model can now be displayed (Fig 12) showing oscillating changes in populations over a period of 1000 time intervals, a task hardly possible without the use of computing technology.

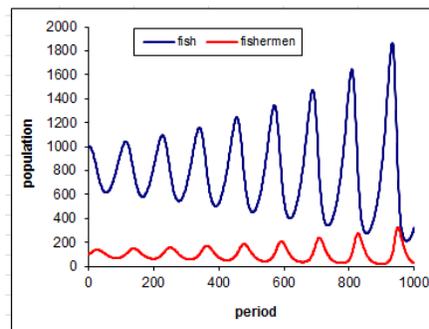


Figure 12 Oscillating changes in the sizes of the two populations over 1000 time intervals.

We can note that corresponding to a maximum point on one curve there exists a minimum point in the other. We can also note a phase shift or time lag between the two curves a minimum in the number of fishermen follows after a minimum of number of available fish as could be expected intuitively. Then when (slightly after) the population of fish increases the population of the fishermen will increase. That means that when the fish population becomes large again, the fishing will resume (after a suitable time lag) and with the fishing activity the fish population will decrease. We note that the wavelength of successive oscillations remains constant over time and that the amplitude of oscillations increases over time.

These outcomes can then lead to interesting discussions with students as to cause and effect. The effects of different input factors can be explored with immediate results, experiments which would not normally be feasible in a real population. Further, simplifying assumptions in the model (a characteristic of all models) could be explored. For example, the model assumes that if the fish population doubles, the number of fish caught per fisherman doubles, which might not always be the case. Also assumed are an unlimited supply of food for the fish population as it grows, and that fishermen are the only predators present thus ignoring the possible presence of sharks hunting for prey in their natural habitat. From all this students could conclude that the results of the model are largely qualitative rather than quantitative. However, they do correspond to what could be intuitively expected.

We now show to the R platform as an alternative and powerful tool for similar modelling experiences.

R platform

The R platform is open source software designed by Ross Ihaka and Robert Gentleman about twenty two years ago and developed by dozens of volunteers (Everitt & Hothorn, 2012). Examples in R textbooks are fully reproducible and all analyses and results, including figures and tables, can be reproduced by the reader. R is designed as a statistical package and in theory that makes users think that it will be used for statistical purposes only, but there are much other functionality available using R.

R's graphical user interface

R has a user friendly graphical user interface (GUI) as shown (Fig 13). When scripting, the scripts can be written on the R console or on an R editor which can be opened from File tab's New Script option.

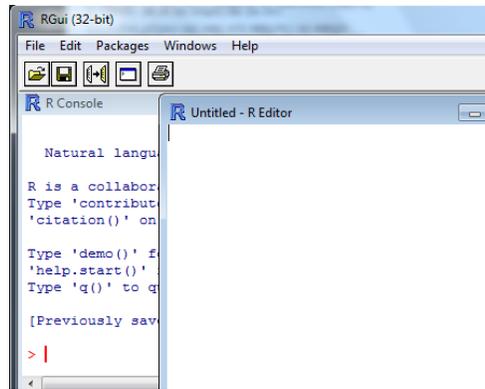


Figure 13 R GUI for scripting

The tabs on R GUI (Fig 14) will help in carrying out many tasks. For example, the File tab provides commands for opening scripts, loading and saving workspace, the Edit tab provides commands such as the “Clear Console” and “Select All”. The “View” tab has two commands, “Toolbar” and “Statusbar”. The “Packages” tab is important tab as it allows different packages for creating different types of graphs and calculations to be downloaded.

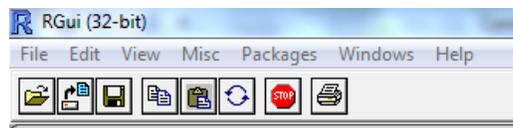


Figure 14 shows the available tools and tabs in R GUI

R software can carry out mathematical calculations so long as a suitable script is written to provide correct instructions for the task to be implemented (Fig 15).

```
> 5+5
[1] 10
```

Figure 15 Simple addition using R script.

Also, data stored in excel can be saved as a .csv file for importing into R and where it can be further displayed and processed. These data can be simply called for display or can be tabled or graphed (Figs 16 & 17).

	A	B
1	Malaria	Asthma
2	10	12
3	15	10
4	20	18
5	13	15
6	17	14
7	20	16

Figure 16 Data is displayed in a .csv file.

```
> getwd()
[1] "D:/Users/dbenig/Docur
> mydata=read.csv(file="2r
> names(mydata)
[1] "Malaria" "Asthma"
```

Figure 17 The data in Fig 15 is read into R.

The `getwd()` function (Fig 17) is used to display the folder in which the .csv file is located and the `mydata=read.csv(file="2.)` line make assigns all data in the .csv file to mydata. `names(mydata)` is the script for outputting the headings of data in the .csv table.

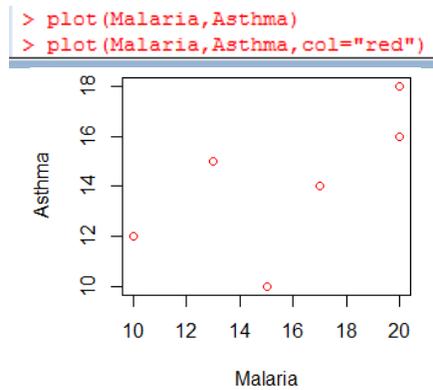


Figure 18 Presentation of a plot/graph of the data in Figure 16.

The R software also allows the formatting of graphs created by adding colours to the graph and widening the thicknesses of planes that's only to name a few. Here (Fig 18) the script line "`plot(Malaria, Asthma, col="red")`" sets the colour of the point circles to red. Further examples of R as an analytical and graphical tool will now be provided.

Using R as an analytical and graphical tool

R as an analytical tool can be used in Linear Algebra for the creating of a matrix and for the calculation of the matrix's determinant (Fig 19).

```

> A=matrix(c(1,6,1,3,2,4,5,0,2),nrow=3,ncol=3)
> A
      [,1] [,2] [,3]
[1,]    1    3    5
[2,]    6    2    0
[3,]    1    4    2
> det(A)
[1] 78
> |

```

Figure 19 R is used in a linear algebra exercise to calculate the determinant of a matrix.

R can also be used as a graphical tool with the ability to allow different libraries to run in it. Thus, it can be used in probability and statistics to produce histograms and normal curves for showing some data and its frequencies. Suitable coding or scripting (Figure 20) can be written to produce a histogram and normal curve (Figure 21).

```

-----, (-----,
hist(Practice,col=gray(.9),xlab="Class Limits",xlim=c(0,100),ylim=c(0,40))
par(mfrow=c(3,3))
hist(Practice)
hist(Practice,
breaks=20,
col="red",
rug(jitter(Practice))
lines(density(Practice), col="blue", lwd=2)
x <- Practice
h<-hist(x,
breaks=20,
col="red",
xlab="Number of students in a class",
main="Histogram with normal curve and box")
xfit<-seq(min(x), max(x), length=40)
yfit<-dnorm(xfit, mean=mean(x), sd=sd(x))
yfit <- yfit*diff(h$mids[1:2])*length(x)
lines(xfit, yfit, col="blue", lwd=2)
box()

```

Figure 20 R scripting to produce the graphing in Figure 20).

The script (Figure 20) has R produce a graph with the following characteristics (Figure 21). The histogram shows 20 red columns along the x-axis. The title of the graph will be “Histogram with normal curve and box”. The x label of the graph is “Number of students in a class and the normal curve with a blue line and a box with the default color black all around the histogram.

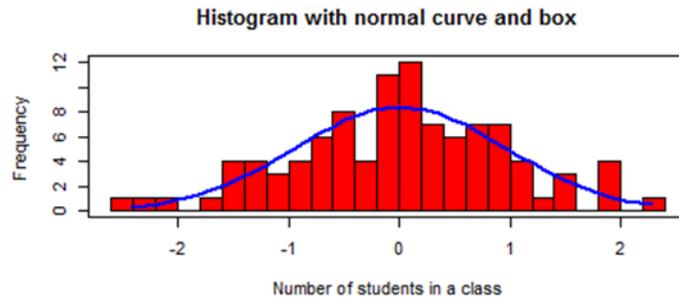


Figure 21 is a display/the output of Fig 20.

The above examples have demonstrated that mathematics at university level can be illustrated, graphed and modelled with various readily available software tools. These have the power to introduce different ideas, subjects and approaches.

The unifying power of mathematics

Not only can mathematical procedures be modeled and illustrated using diverse available software as shown in this paper, but a given mathematical procedure can have a wide range of modelling applications. Various authors of mathematics text books (eg. Krezsig, 2011) emphasise for students the value of basic mathematical concepts, principles and procedures in explaining and Modeling a wide range of applications. A single mathematical equation can be applied to a wide range of applications.

“Entirely different physical systems may have the same mathematical model and thus may be treated by the same mathematical methods”.
(Krezsig 2011, p. 566)

An example often used is the second order linear ordinary differential equation Modeling vibrations in nature, machinery or musical instruments. This will be the subject of a forthcoming paper in this series on the use of mathematical Modeling both to explore real world situations and to motivate students to appreciate the power and value of mathematics.

Summary and conclusion

This paper has proposed Modeling and display of mathematical procedures as a means for motivating university students to learn and enjoy mathematics, as well, of course, for explicating mathematical processes. It has proposed spreadsheets as a readily available and powerful tool for Modeling mathematics. It has also demonstrated with simple examples the use of the freely available R platform as a scripting language to demonstrate and graph a wide range of mathematical procedures.

Similarly it has further noted the applicability of a single mathematical procedure to a wide variety of real world appreciations giving rise to the appreciation of the unifying power of mathematics.

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